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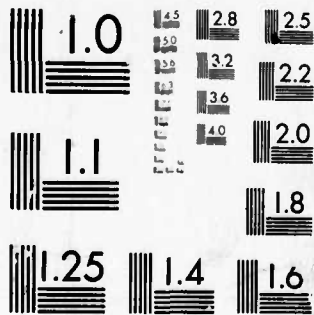
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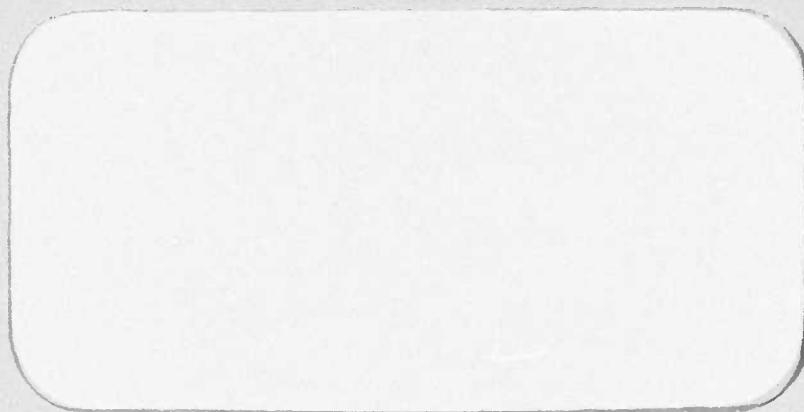
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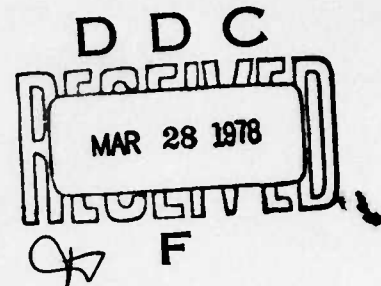
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PERTURBATION SOLUTIONS FOR
VARIABLE ENERGY BLAST WAVES



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Edward T. Pitkin

ABSTRACT

The trajectory of and the flow field behind blast waves with time varying energy input is determined. Freeman's² Lagrangean coordinate formulation is modified to include both the geometric factor, α , for plane, cylindrical and spherical shocks and also non-integer values of β , the energy input parameter, in a single computational algorithm. Numerical problems associated with vanishing density at the fictitious piston face are then examined and solved. Second order perturbation solutions about the infinite strength shock are then obtained in Sakurai's¹ inverse shock Mach number expansion parameter for $0 \leq \beta < \alpha + 1$. Tables and graphs of significant numerical coefficients are presented for comparison to and extension of results of other authors. Graphs of typical shock trajectories and flow field density, pressure and velocity variations are also presented and discussed.

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PERTURBATION SOLUTIONS FOR VARIABLE ENERGY BLAST WAVES

Edward T. Pitkin†

1. INTRODUCTION

The analytical theory of blast waves has been the subject of intensive study by many investigators since the pioneering work of G. I. Taylor³ about thirty years ago. A large fraction of this work concerns waves of constant, instantaneously supplied energy which propagate and decay according to the dictates of inviscid isentropic flow behind the wave and Rankine-Hugoniot conditions at the wave front. This work has been well summarized by Sakurai¹ who has also developed an efficient method of expanding the general solution about the simpler limiting solution for the flow behind a very strong shock. Furthermore, he has shown that the square of the inverse shock Mach number is a most convenient and natural parameter to use for this expansion.

A lesser amount of work has been devoted to analysis of blast waves of variable energy input. In 1957 Lees and Kubota⁶ examined this case briefly in connection with the hypersonic blast wave analogy and Rogers³ presented the limiting strong shock solution in 1958. More recently in 1968 Freeman² examined this case in connection with cylindrical spark channel formation from exploding wires and Dabora⁷ has applied variable blast wave techniques to droplet explosions in spray detonations.

The work reported here is basically an extension of Freeman's approach to cover the full spectrum of plane, cylindrical and spherical variable

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energy blast waves within a single analytic formulation and computation algorithm. It also contains corrections to some numerical errors discovered in Freeman's paper. As a consequence Freeman's notation has been adopted completely and derivations given in his paper have only been summarized here. His major contribution was to recast Sakurai's expansion procedure in Lagrangean coordinates, replacing distance from the origin, r , with mass, m , as an independent variable, but retaining the inverse shock Mach number as the expansion parameter. The advantage is two fold: (1) the number of differential equations that must ultimately be integrated numerically is reduced by nearly a factor of two, and (2) the position of the fictitious moving piston face which supplies the variable energy is uniquely defined to be at $mass = 0$. In Eulerian coordinates precise determination of the piston position can be a very difficult numerical task.

In sections 2 and 3, the equations of motion and the strong shock solution are reviewed. The expansion procedure is outlined in section 4 and a method of extrapolating the numerical solution to the mass origin is discussed in section 5. Numerical results are then summarized and discussed in section 6.

2. EQUATIONS OF MOTION

Let p_0 and ρ_0 be the pressure and density in the undisturbed medium while r is the radius from the origin to an arbitrary point in space and M is the total mass contained between the origin and r . Let $r = R$ at the shock front so that the volume behind the shock front, when filled with gas of original density, will contain the mass

$$M_0 = 2^\alpha \pi^{\alpha(3-\alpha)/2} R^{\alpha+1} \rho_0 / (\alpha+1) \quad (2.1)$$

where α is a geometric parameter with values 0, 1, and 2 for plane, cylindrical and spherical waves respectively. Finally let u be the speed at any point and U be the speed of the shock front, C the speed of sound and γ the specific heat ratio in the undisturbed gas.

It is convenient to introduce non-dimensional variables. For mass, radius, velocity and density take

$$m \triangleq M/M_0 \quad (2.2)$$

$$a \triangleq r/R \quad (2.3)$$

$$f \triangleq u/U \quad (2.4)$$

$$h \triangleq \rho/\rho_0 \quad (2.5)$$

while the square of the inverse shock Mach number

$$y \triangleq C^2/U^2 = C^2/(dR/dt)^2 \quad (2.6)$$

may be combined with p_0 to define a variable related to non-dimensional pressure

$$g = py/p_0 \quad (2.7)$$

An arbitrary non-zero initial radius of the shock, R_0 , is then used to define

a characteristic time

$$t_0 \triangleq R_0/C \quad (2.8)$$

which may then be used to non-dimensionalize time through

$$\omega \triangleq t/t_0 \quad (2.9)$$

while the shock front coordinate, R , is non-dimensionalized by

$$z \triangleq R/R_0 \quad (2.10)$$

Finally the decay index, a ratio of the percentage changes of shock Mach number and radius, as introduced by Sakurai¹, is

$$\lambda = \frac{(dy/y)}{(dz/z)} = \frac{-2Rd^2R/dt^2}{(dR/dt)^2}, \quad (2.11)$$

the latter obtained by use of (2.6) and (2.10).

The equations of fluid flow behind the shock wave have been derived in Lagrangean coordinates by Freeman². Only the results will therefore be summarized here. First the density at any point is given by

$$h = \rho/\rho_0 = 1/[(\alpha+1)a^\alpha \partial a/\partial m] \quad (2.12)$$

and the non-dimensional velocity is

$$f = a - (\alpha+1)m \frac{\partial a}{\partial m} + \lambda y \frac{\partial a}{\partial y}. \quad (2.13)$$

Beginning with the assumption of a purely isentropic process behind the shock for each mass element it follows that

$$\partial(p/\rho^\gamma)/\partial y = 0 \quad (2.14)$$

which upon using the non-dimensional variables above can be shown to lead to the following in Lagrangean coordinates,

$$\begin{aligned}
 (\alpha+1)m \left[\left(\frac{\alpha}{a} \frac{\partial a}{\partial m} + \frac{1}{\gamma g} \frac{\partial g}{\partial m} \right) \frac{\partial a}{\partial m} + \frac{\partial^2 a}{\partial m^2} \right] + \frac{\lambda}{\gamma} \frac{\partial a}{\partial m} \\
 - \lambda y \left[\left(\frac{\alpha}{a} \frac{\partial a}{\partial y} + \frac{1}{\gamma g} \frac{\partial g}{\partial y} \right) \frac{\partial a}{\partial m} + \frac{\partial^2 a}{\partial m \partial y} \right] = 0 .
 \end{aligned} \tag{2.15}$$

Similarly Newton's second law for the mass element

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial y} \frac{dy}{dt} = -(2r)^\alpha \pi^{\alpha(3-\alpha)/2} \frac{\partial p}{\partial m} \tag{2.16}$$

leads to

$$\begin{aligned}
 \frac{a^\alpha (\alpha+1)}{\gamma} \frac{\partial g}{\partial m} - \frac{\lambda a}{2} + (\alpha+1) \frac{2\alpha+\lambda}{2} m \frac{\partial a}{\partial m} + (\alpha+1)^2 m^2 \frac{\partial^2 a}{\partial m^2} \\
 + \lambda y \left[-2(\alpha+1)m \frac{\partial^2 a}{\partial m \partial y} + \frac{2+\lambda}{2} \frac{\partial a}{\partial y} + y \frac{\partial a}{\partial y} \frac{\partial \lambda}{\partial y} + \lambda y \frac{\partial^2 a}{\partial y^2} \right] = 0 .
 \end{aligned} \tag{2.17}$$

The energy contained between the origin and the shock is the sum of the kinetic energy and the internal energy. The increment of energy over that of the same volume of undisturbed gas is

$$E = \int_0^{M_0} \left(\frac{u^2}{2} + \frac{p}{(\gamma-1)\rho} - \frac{p_0}{(\gamma-1)\rho_0} \right) dm . \tag{2.18}$$

Converting to non-dimensional form, using Equation (2.12) and letting a prime designate $\partial/\partial m$ gives

$$\frac{E_{p_0}}{(\alpha+1)p_0 M_0} = \frac{1}{y} \int_0^1 \left(\frac{\gamma f^2}{2(\alpha+1)} + \frac{g a^\alpha a'}{\gamma-1} \right) dm - \frac{1}{(\gamma-1)(\alpha+1)} \quad (2.19)$$

Letting $b = 1/[(\gamma-1)(\alpha+1)]$, $E_\alpha = E/[2\pi^{(3-\alpha)/2}]^\alpha$ and designating the integral J after Sakurai¹ this becomes

$$\frac{E_\alpha}{p_0 R^{\alpha+1}} = \frac{J}{y} - b \quad (2.20)$$

If the total energy behind the shock varies as a power of time according to the following rule

$$E_\alpha = R_0^{\alpha+1} p_0 (t/t_0)^\beta, \quad (2.21)$$

the energy equation reduces to the simple form

$$\omega^\beta / z^{\alpha+1} = J/y - b. \quad (2.22)$$

When $\beta = 0$ the energy is constant, a case examined by many investigators and well documented by Sakurai. Letting β take non-zero values allows one to model a fairly wide range of energy inputs which can be further extended with some modification of (2.21) as has been shown by Freeman².

3. STRONG SHOCK LIMIT

At very large shock Mach numbers $y \rightarrow 0$ and $J/y \gg b$ so the energy equation reduces to a simple form consistent with Rogers'³ similarity solution for the equation of motion for which the shock front advances according to

$$R = \left[R_0^{\alpha+1} C^2 / n^2 J_0 t_0^\beta \right]^{1/(\alpha+3)} t^n = K t^n \quad (3.1)$$

where n is related to β , α and λ through

$$n = (2+\beta)/(\alpha+3) = 2/(\lambda_0+2) \quad (3.2)$$

from which the decay index for strong shocks is

$$\lambda_0 = 2(\alpha+1-\beta)/(2+\beta) \quad (3.3)$$

The zero subscripts on λ and J are used to designate the strong shock case here.

Note also that

$$\frac{dz}{d\omega} = \frac{t_0}{R_0} \frac{dR}{dt} = \frac{U}{C} = \frac{1}{y^{1/2}} \quad (3.4)$$

while from (3.1)

$$dz/d\omega = nz/\omega \quad (3.5)$$

so that

$$z = \omega / n y^{1/2} \quad (3.6)$$

Combining this with the strong shock energy equation ($b = 0$) yields

$$z = z_0 y^{1/\lambda_0} \quad (3.7)$$

where

$$z_0 = \left(n^\beta / J_0 \right)^{1/(\alpha+1-\beta)} \quad (3.8)$$

4. OUTLINE OF SERIES SOLUTION

Equations (2.15) (2.17) and (2.22) comprise a system of three dependent variables; λ , a , and g and two independent variables; m and y . Sakurai has shown that λ is a function of y only and that the pressure, density etc. are slowly varying functions of y . He therefore suggests expansion in powers of y to reduce the number of independent variables. In the Lagrangean coordinates used here this has the very desirable effect of reducing the partial differential equations to ordinary differential equations.

The expansions, taken about the strong shock solution for which $y = 0$, are

$$\lambda(y) = \lambda_0 + \lambda_1 y + \lambda_2 y^2 + \dots \quad (4.1)$$

$$a(m,y) = a_0(m) + a_1(m)y + a_2(m)y^2 + \dots \quad (4.2)$$

$$g(m,y) = g_0(m) + g_1(m)y + g_2(m)y^2 + \dots \quad (4.3)$$

By similar reasoning the equation for the shock front is expanded about the strong shock "zero order" solution (3.7) as

$$z = z_0 y^{1/\lambda_0} (1 + z_1 y + z_2 y^2 + \dots) \quad (4.4)$$

Equation (3.4) which applies in general can be integrated to give:

$$\omega = \int y^{1/2} \frac{dz}{dy} dy \quad .$$

Substituting for z and taking $z = 0$ at $\omega = 0$ then gives an expansion for the time variable as a function of y and z

$$\omega = z_0 y^{(\lambda_0+2)/2\lambda_0} (\omega_0 + \omega_1 z_1 y + \dots + \omega_j z_j y^j + \dots) \quad (4.6)$$

where

$$\omega_j = (2j\lambda_0+2)/((2j+1)\lambda_0+2) \quad (4.7)$$

Noting from (2.13) that $f = f(\lambda, a, m)$ it is clear that substitution of (4.1 - 4.3) into (2.19) will yield integrals in ascending powers of y so that J can be expressed as

$$J = J_0 + J_1 y + J_2 y^2 + \dots \quad (4.8)$$

These expansions are then substituted into the isentropic, momentum and energy equations (2.15), (2.17) and (2.22) and the coefficients of like powers of y equated to give perturbation equations of ascending order. In the isentropic and momentum equations this procedure yields ordinary differential equations for a_j and g_j with λ_j entering as a parameter. These must be solved subject to the shock jump boundary conditions.

$$\begin{aligned} a_0(1) &= 1 & a_1(1) &= 0 \\ g_0(1) &= 2\gamma/(\gamma+1) & g_1(1) &= (1-\gamma)/(\gamma+1) \\ a'_0(1) &= \frac{\gamma-1}{(\alpha+1)(\gamma+1)} & a'_1(1) &= \frac{2}{(\alpha+1)(\gamma+1)} \end{aligned} \quad (4.9)$$

The remaining coefficients are all zero at the shock. Fortunately the successive orders of approximation to the isentropic equation yield intermediate integrals for g_j . These may then be used in the momentum equation which must be solved by numerical integration from the shock front to the mass

origin. The equations and solutions for each order will be discussed in the following sections.

Now consider the energy equation. Putting the expansion (4.4), (4.6) and (4.8) into (2.22) gives

$$\begin{aligned} & z_0^\beta (\omega_0 + \omega_1 z_1 y + \omega_2 z_2 y^2 + \dots)^\beta \\ & = z_0^{\alpha+1} (1 + z_1 y + z_2 y^2 + \dots)^{\alpha+1} (J_0 + (J_1 - b)y + J_2 y^2 + \dots). \end{aligned} \quad (4.10)$$

The two series with exponents may be converted to simple power series by means of the binomial formula if the first term of each is larger than the sum of the remaining terms. This is certainly true for small y . Multiplying out on the right and equating coefficients of like powers of y then yields relations between J_j and z_j :

$$\begin{aligned} J_0 &= \omega_0^\beta z_0^{\beta-(\alpha+1)}, \\ J_1/J_0 &= \left[\beta \omega_1 / \omega_0 - (\alpha+1) \right] z_1 + b, \\ J_2/J_0 &= \left[\beta \frac{(\beta-1)}{2} \frac{\omega_1^2}{\omega_0^2} + (\alpha+1) \left(\frac{\alpha+2}{2} - \frac{\beta \omega_1}{\omega_0} \right) \right] z_1^2 \\ &\quad + \left[\frac{\beta \omega_2}{\omega_0} - (\alpha+1) \right] z_2. \end{aligned} \quad (4.11)$$

These relations are equally valid for integer and non integer values of β . Freeman² only considered integer values. Substituting (4.4) into (2.11) and solving for the z_j gives

$$z_1 = -\lambda_1/\lambda_0^2$$

$$z_2 = -[\lambda_2 + (1+\lambda_0)z_1\lambda_1]/2\lambda_0^2$$

$$\begin{aligned} z_3 &= -[\lambda_3 + (1+\lambda_0)z_1\lambda_2 + (1+2\lambda_0)z_2\lambda_1]/3\lambda_0^2 \\ &\vdots \end{aligned} \tag{4.12}$$

which may then be used in (4.11) to give relations between the λ_j and J_j .

These will be used later to evaluate the λ_j for each order of approximation.

The solution proceeds in ascending orders of approximation. The zeroth order, corresponding to the strong shock solution with $y \rightarrow 0$, is used in the computation of the first order solution and the first order solution is then used to compute the second and so forth. In this paper the solution will be carried only to the 2nd order.

The zeroth order approximation to the isentropic equation is

$$\frac{g_0'}{g_0} = -\left[\frac{\lambda_0}{(\alpha+1)m} + \gamma\left(\alpha \frac{a_0'}{a_0} + \frac{a_0''}{a_0^2}\right)\right] \tag{4.13}$$

which Freeman has shown to have an integral

$$g_0 = K_0 / (a_0^\alpha a_0')^\gamma m^{\lambda_0/(\alpha+1)} \tag{4.14}$$

where

$$K_0 = \frac{2\gamma}{\gamma+1} \left[\frac{\gamma-1}{(\alpha+1)(\gamma+1)} \right]^\gamma \tag{4.15}$$

The momentum equation in the zeroth approximation becomes

$$a''_0 = (B_0 a'_0 - C_0 a_0 - g_0 a_0^\alpha \lambda_0 / m) / A_0, \quad (4.16)$$

where

$$A_0 = (\alpha+1) \gamma [a_0^\alpha g_0 / a'_0 - (\alpha+1) m^2],$$

$$B_0 = (\alpha+1) m \gamma (\alpha + \lambda_0 / 2),$$

$$C_0 = \gamma [\lambda_0 / 2 + \alpha(\alpha+1) g_0 a_0^\alpha a'_0 / a_0^2]. \quad (4.17)$$

The first approximation to the energy integral, J , of (4.8) will be obtained by evaluating

$$J_0 = \int_0^1 \left[\frac{\gamma}{2} \left(\frac{a_0 - (\alpha+1) m a'_0}{\alpha+1} \right)^2 + \frac{g_0 a_0^\alpha a'_0}{\gamma-1} \right] dm \quad (4.18)$$

along with integration of (4.16) from the shock front to the mass origin.

(Note that Freeman's forms of this and higher order integrals are only correct for $\alpha = 1$.) The first term of this expression is a measure of the kinetic energy behind the shock and the second term is proportional to the internal energy so these two factors can easily be accumulated separately. The integration of a_0 is initiated with the boundary conditions (4.9). The values of $a_0(m)$, $a'_0(m)$, $g_0(m)$ and $g'_0(m)$ may be stored for use in the first order solution or, more conveniently, these integrations can be performed simultaneously with those for the higher order solutions.

In the zeroth order equation λ_0 appears as a known parameter obtained from the similarity solution. In the higher order equations the λ_j 's appear as unknown parameters. Fortunately, the equations are linear in λ_j and of course the perturbation equations are linear differential equations so it follows that

superposition of solutions is valid. This property will allow evaluation of the λ_j 's after the equations have been integrated.

In each higher order approximation the g_j' term in the momentum equation may be eliminated with the isentropic equation and its integral, g_j . The integrals have been obtained through third order by Freeman. The momentum equation then gives higher order perturbation equations of the form

$$a_j'' = (B_j a_j' + C_j a_j + D_j + \lambda_j E_j)/A_0 \quad (4.19)$$

in which the coefficients B_j through E_j are functions only of known quantities from lower order solutions. It follows that a_j can be expressed as the linear combination

$$a_j = a_{j1} + \lambda_j a_{j2} \quad (4.20)$$

so that (4.19) gives two equations

$$\begin{aligned} a_{j1}'' &= (B_j a_{j1}' + C_j a_{j1} + D_j)/A_0, \\ a_{j2}'' &= (B_j a_{j2}' + C_j a_{j2} + E_j)/A_0. \end{aligned} \quad (4.21)$$

It is convenient to let the non-zero boundary condition of (4.2) be satisfied by a_{11} so the remaining perturbations are zero at the shock. Note that the denominator, A_0 , is the same for all orders of approximation. This term goes to zero at $m = 0$ when $\beta = 0$ causing computational difficulties which will be discussed later.

It will be evident upon substitution of a_j into the isentropic equation that g_j must be of the form

$$g_j = g_{j1} + \lambda_j g_{j2}. \quad (4.22)$$

Finally substituting a_j and g_j into the energy integral will show J_j to be of the form

$$J_j = J_{j1} + \lambda_j J_{j2} = \int_0^1 (J'_{j1} + \lambda_j J'_{j2}) dm \quad (4.23)$$

where the terms J_j , and J_{j2} are to be accumulated simultaneously with integration of (4.21). When J_{j1} and J_{j2} have thus been evaluated λ_j can be determined. After using (4.12) in (4.11) to eliminate z_j in favor of λ_j , (4.23) is substituted in to the left hand side of (4.11) and the result is then solved for λ_j . The first two of these coefficients are

$$\lambda_1 = \frac{b - J_{11}}{J_{12} + J_0[\omega_1\beta/\omega_0 - (\alpha+1)]/\lambda_0^2} \quad (4.24)$$

and

$$\lambda_2 = \frac{J_0 \lambda_1^2 G - 2\lambda_0^2 J_{21}}{J_0 H - 2\lambda_0^2 J_{22}} \quad (4.25)$$

where

$$\begin{aligned} G &= \beta(\beta-1) \omega_1^2/\omega_0^2 + (\alpha+1)(\alpha+1-\lambda_0-2\beta\omega_1/\omega_0) \\ &\quad + (1+\lambda_0) \beta \omega_2/\omega_0 \\ H &= \beta(\omega_2/\omega_0) - (\alpha+1) \end{aligned} \quad (4.26)$$

The ω_j are obtained from (4.7). Expressions for the coefficients A_j through E_j , J'_{j1} and J'_{j2} are listed in the Appendix.

Freeman has shown that given approximations through the k 'th order with $k \geq 2$, it is possible to obtain an excellent approximation to the shock position $R(t)$ as follows. Truncate the λ series at the $k+2$ term and evaluate the

last two λ_j in terms of those preceeding and known boundaries of the λ vs y curve at $y = 1$. Thus

$$\lambda(1) = \lambda_0 + \lambda_1 + \dots + \lambda_k + \lambda_{k+1} + \lambda_{k+2} = 0 \quad (4.27)$$

$$\begin{aligned} d\lambda(1)/dy &= \lambda_1 + 2\lambda_2 + \dots + k\lambda_k + (k+1)\lambda_{k+1} + (k+2)\lambda_{k+2} \\ &= (2+\alpha)/4 \end{aligned} \quad (4.28)$$

The higher order λ terms in (4.12) are then zero and all the z_j can be expressed in terms of λ_0 through λ_{k+2} . If $k = 2$ the general expression for z_j when $j \geq 5$ is

$$\begin{aligned} z_j = - \left([1+(j-4)\lambda_0]\lambda_4 z_{j-4} + [1+(j-3)\lambda_0]\lambda_3 z_{j-3} \right. \\ \left. + [1+(j-2)\lambda_0]\lambda_2 z_{j-2} + [1+(j-1)\lambda_0]\lambda_1 z_{j-1} \right) / j\lambda_0^2 \end{aligned} \quad (4.29)$$

Using this and (4.12) in (4.4) and (4.6) then gives $z(\omega)$ or $R(t)$ for all values of y .

It was found that this procedure works very well for $\beta = 0$, but when β approaches $\alpha + 1$ where $\lambda \equiv 0$, satisfaction of the final slope condition (4.28) with a limited number of terms causes a false hump in the λ - y curve near $y = 1$. The procedure was therefore modified through multiplying λ_{k+2} as obtained by simultaneous solution of (4.27) and (4.28) by the factor $[1-\beta/(\alpha+1)]$ and solving (4.27) for λ_{k+1} once again. Thus the need to meet the final slope conditions is progressively eliminated as β approaches $\alpha + 1$ to give more realistic λ - y curves.

5. EXTRAPOLATION TO $m = 0$.

When $\beta = 0$ as it does in the constant energy blast wave, integration of (4.16) will show that $a'_0 \rightarrow \infty$ and $a_0 \rightarrow 0$ as $m \rightarrow 0$ so it follows from (4.17) that $A_0 \rightarrow 0$. This latter term appears as the denominator of (4.21) so it may be anticipated that computational difficulties will be encountered near the mass origin for all orders of approximation. The problem is further compounded in evaluation of the energy integrals which contain terms proportional to a'_0 and accumulate rapidly near $m = 0$. For example if $\gamma = 1.1$ and $\alpha = 1$, sixty percent of the true value of J_0 is accumulated in the last one percent of mass variation. Numerical integration all the way to the origin is clearly impossible yet it is mandatory that accurate values of J_j be obtained else the next higher order approximation be seriously in error.

Freeman² mentioned this problem and suggested an "appropriate approximation formula" but gave no details. Similar numerical problems arising in the Eulerian formation of the problem have been noted by Sakurai¹ and Bach and Lee⁴. The procedure outlined here was developed by the author and is based upon the fact that in a region close to the mass origin the pressure term, g_0 , can be taken to be constant to a very high degree of approximation. Although the procedure is only necessary when β is close to zero, it is still valid for larger values of β so it can be incorporated into a computer program valid for all values of β .

Consider the integrand of (4.18) in which the first term is related to the kinetic energy and the second to the internal energy.

$$J'_0 = \frac{\gamma}{2(\alpha+1)} (a_0 - (\alpha+1)ma'_0)^2 - \frac{g_0 a_0^\alpha a'_0}{\gamma-1} \quad (5.1)$$

In the region near the origin, say $m < 10^{-3}$, the density is very low so it may be anticipated - and verified by numerical experiment - that the first

term is much smaller than the second. Furthermore it has been observed from numerical integrations that the product ma'_0 approaches zero at nearly the same rate as a_0 so that this first term is nearly constant. These numerical integrations also showed g_0 to be constant to about four significant figures in this region. It follows that taking these factors constant over an interval of $\Delta m \leq 10^{-3}$ will introduce no significant error in the computation of J_0 .

The major contributing factor to the integral is then $a_0^\alpha a'_0$ which by use of (4.14) can be expressed in terms of m as

$$a_0^\alpha a'_0 = (K_0/g_0)^{1/\gamma} m^{-\lambda_0/\gamma(\alpha+1)} \quad (5.2)$$

which can be integrated analytically if g_0 is taken to be constant.

The procedure then is to integrate numerically from $m = 1$ to $m = \delta$ where $0 < \delta \leq 10^{-3}$ and add a correction term obtained by analytical integration from 0 to δ , i.e.,

$$\Delta J_0 = \frac{\gamma \delta}{2(\alpha+1)} (a_0 - (\alpha+1)\delta a'_0)^2 + \frac{g_0}{\gamma-1} \left(\frac{K_0}{g_0} \right)^{1/\gamma} \left[\frac{\delta^{1-\lambda_0/\gamma(\alpha+1)}}{1-\lambda_0/\gamma(\alpha+1)} \right] \quad (5.3)$$

where a_0 and a'_0 are evaluated at $m = \delta$.

Similar considerations apply to the higher order approximations to J . In each case the kinetic energy term in the integrand can be taken as constant over the last integration interval while the internal energy term can be expressed in terms of $a_0^\alpha a'_0$, g_0 and essentially constant ratios such as a_j/a_0 , a'_j/a'_0 , etc. This method has been checked by letting δ take on values from 10^{-3} to 10^{-6} and noting that all give the same final values for each J_j . A value of $\delta = 10^{-5}$ was used in the computations leading to the results reported in the next section. Formulas for the correction term ΔJ_{11} , ΔJ_{12} etc. are given in the appendix.

6. RESULTS

The equations outlined in Sections 3, 4 and the Appendix have been numerically integrated with a fourth order Runge-Kutta algorithm in double precision and extrapolated to $m = 0$ as discussed in Section 3. The procedure has been carried out to the second order in the expansion variable y (some authors^{1,2,4} prefer "third approximation") for plane, cylindrical and spherical blast waves, i.e., for $\alpha = 0, 1, 2$. The range $0 \leq \beta < \alpha + 1$ (above which the zeroth order similarity solution ceases to exist⁶) was covered for $\gamma = 1.1$ and 1.4 . The solution for the case, $\beta = \alpha + 1$, which corresponds to the well known self similar solution for which $\lambda = 0$ and $R = Kt$, is computationally unobtainable because of a zero divisor problems evident in equation 3.7. However, solutions for β very close to this value (within .01) were obtained easily. Solutions have also been obtained for other typical values of γ for integer values of β .

Tables 1 to 3 are summaries of the expansion coefficients λ_1 , λ_2 and J_0 , J_1 , J_2 obtained in the computation. Korobienikov and Chushkin⁵, Sakurai¹, Bach and Lee⁴ have all published values of λ_1 computed in Eulerian coordinates for $\beta = 0$, $\gamma = 1.4$ which agree with those given here. (Note that Bach and Lee report values of $\lambda_1/2$ while the others give values of $\lambda_1/(\alpha+1)$). The values of λ_2 and λ_1 for $\beta = 0$ and $\gamma = 1.4$ are also in agreement to the full six significant figures quoted by Bach and Lee. The values of λ_1 for $\beta = 0$ and $\gamma = 1.1$ to 3.0 are in agreement with those reported by Freeman, however his values of λ_1 for $\beta = 1$ are in error because he apparently used an incorrect value of λ_0 to obtain them. His values of J_{11} and J_{12} are, however, correct. As a final check on numerical accuracy it is noted that the values of J_0 reported to 4 significant figures by Rogers for $\gamma = 1.2$ and 1.4 and various β agree with values in these tables.

It can be seen from the tables that the λ coefficients approach zero

rapidly as β is increased from zero. This is also shown in Figure 1. The maximum value of β is $\alpha+1$ where $\lambda = 0$ and the shock propagates with constant velocity. This drop off of the decay index implies much stronger convergence of the perturbation series as β increases.

Figure 2 shows typical variations of the J coefficients with β for $\alpha = 2$, and $\gamma = 1.4$. It is seen that J_2 drops off rapidly and J_0 becomes fairly constant while J_1 takes on an increasing portion of the energy as β increases. Similar results obtain for other γ and α .

Radial pressure, density and velocity distributions are presented in Figures 3 to 5 for $\alpha = 2$, $\gamma = 1.4$ and $\beta = 0, 1, 2$. The hump in the density distribution for $\gamma = .5$ and $\beta = 0$ is most likely due to truncation of the expansion at the third term. This suggests an upper limit of validity for $\gamma = .5$, or a shock Mach number of 1.4, for the second order expression when $\beta = 0$. This limit can be raised considerably as β increases as is evidenced by the curves for $\beta = 1, 2$.

As β is increased from zero, the constant energy case, the flow region rapidly contracts to a thin shell between the shock and the fictitious piston located at the zero mass position as is evident in Figures 4 and 5. Even when $\beta = 0$, over 99% of the mass is located in the outer half radius of the sphere but as noted before, over 50% of the total energy can be confined to the inner 1% of the mass because the temperature becomes extremely large in this region. As β approaches $\alpha+1$ the solution approaches the self similar result for $\lambda = 0$ and $\gamma = \text{constant}$. At this limit the spherical blast wave flow field occupies a thin shell within which the velocity, pressure and density are constant. Similar results obtain for other γ and α . In particular the profiles for $\alpha = 1$, $\beta = 1$ and $\gamma = 1.4$ are given in Figure 6 as replacements for similar plots given by Freeman which, as noted before, are in error due to his use of

the incorrect value of λ .

The variation of the decay index, λ , and the shock position, z , with shock strength, y , is given in Figure 7 for a spherical shock with $\gamma = 1.4$ and $\beta = 0$. The decay index was obtained by use of accurate values of λ_0 , λ_1 and λ_2 given in Table 2 and approximate values of λ_3 and λ_4 obtained by matching boundary conditions at $y = 1$ as outlined in section 4. The shock position was obtained from (4.6) with z_j from (4.12) and enough terms of (4.28) to insure that no truncation error would occur in the sixth significant figure. These results are compared with a decay index and shock position obtained from tables of shock position and over-pressure reported by Goldstine and Von Neumann⁸ who solved the exact partial differential equations by a numerical technique. The comparison is most favorable over the whole range of y even though the internal structure of the flow field can only be predicted to $y \approx .5$ for $\beta = 0$. Of course when β becomes larger the internal approximations improve for larger y .

The effect of β on decay index and shock position is given in Figures 8 and 9 where λ and z are plotted against shock strength with β as a parameter. Note the smooth approach to $\lambda = 0$ as β approaches $\alpha+1$ in Figure 8.

It is evident in Figure 9 that a constant value of shock strength is approached in the limit $\beta = \alpha+1$ or $\beta = 3$ in this case. The value can easily be obtained by noting that a solution of the form $z \sim \omega^n$ is required in which $n = (2+\beta)/(\alpha+3) = 1$ to be consistent with the rest of the family of curves. Differentiating this relation and using (3.4), the definition of y , gives

$$dz/d\omega = z/\omega = y^{-1/2} \quad (6.1)$$

while the energy equation (2.22) gives

$$(\omega/z)^{\alpha+1} = Jy - 1/[(\alpha+1)(\gamma-1)] \quad (6.2)$$

Eliminating w/z and using the second order approximation for J consistent with the other curves then yields the following equation which can be solved for the limiting value of y .

$$y^{(\alpha+3)/2} = J_0 + \left(J_1 - 1/[(\alpha+1)(\gamma-1)] \right) y + J_2 y^2 \quad (6.3)$$

Taking $\gamma = 1.4$, the limits are $y = 0.531, 0.528$ and 0.542 for $\alpha = 2, 1$ and 0 , respectively.

The shock trajectories corresponding to Figures 8 and 9 are given in Figure 10. It is clear that there is relatively little difference in trajectories as β is varied. The major effect of increasing β is the faster decay of shock strength at low z and slower decay at large z previously shown in Figure 9. In addition there appears to be only a very weak dependence of the trajectory upon γ as the curves for $\gamma = 1.1$ fall in the same narrow band indicated for $\gamma = 1.4$. The γ effect is more evident in the z - y plots of Figure 11 which also indicate the effect of α for $\beta = 0.5$ in all cases.

The apportionment of energy between kinetic and internal modes is also of interest. These two contributions can easily be separated in the computation of the energy integral, J . The fraction of the total energy associated with kinetic energy is plotted in Figure 12 versus the shock strength with β as a parameter. These results were computed with a three term approximation of J . Note that the kinetic energy fraction increases with β while in all cases most of the energy soon is transformed to internal energy as the shock decays and y increases. The values obtained for $y = 0$ are in agreement with computations made by Rogers³.

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| β | γ | J_0 | λ_1 | J_1 | λ_2 | J_2 |
|---------|----------|---------|-------------|----------|-------------|----------|
| 0 | 1.1 | 2.80158 | -4.17316 | -0.84575 | 5.91593 | 4.14349 |
| | 1.2 | 1.53186 | -4.08477 | -0.62865 | 5.09973 | 1.95302 |
| | 1.3 | 1.09904 | -4.01846 | -0.54156 | 4.57118 | 1.25598 |
| | 1.4 | 0.87708 | -3.96712 | -0.48974 | 4.20319 | 0.92163 |
| | 5/3 | 0.59873 | -3.87486 | -0.40999 | 3.61729 | 0.54144 |
| | 2 | 0.44810 | -3.80867 | -0.35333 | 3.24624 | 0.36366 |
| | 3 | 0.27579 | -3.72635 | -0.26385 | 2.82464 | 0.19475 |
| .25 | 1.4 | 0.70204 | -2.65379 | -0.02745 | 2.18103 | 0.57057 |
| .5 | 1.1 | 0.78106 | -1.79666 | 3.66353 | 1.33644 | 0.59187 |
| | 1.4 | 0.62794 | -1.72133 | 0.22057 | 0.96355 | 0.36104 |
| 1.0 | 1.1 | 0.60523 | -0.65358 | 4.20887 | 0.07644 | 0.15244 |
| | 1.2 | 0.58859 | -0.65324 | 1.73102 | 0.06742 | 0.14316 |
| | 1.3 | 0.54713 | -0.65407 | 0.91562 | 0.06269 | 0.13736 |
| | 1.4 | 0.56127 | -0.65559 | 0.51408 | 0.06044 | 0.13357 |
| | 5/3 | 0.53222 | -0.66114 | 0.04626 | 0.06044 | 0.12830 |
| | 2 | 0.50269 | -0.66886 | -0.17246 | 0.06530 | 0.12565 |
| | 3 | 0.43705 | -0.68943 | -0.35263 | 0.08495 | 0.12245 |
| 1.5 | 1.1 | 0.53960 | -0.18017 | 4.45556 | -0.05095 | 0.02116 |
| | 1.4 | 0.53039 | -0.18812 | 0.69124 | -0.04452 | 0.03804 |
| 1.95 | 1.4 | 0.51401 | -0.01130 | 0.80246 | -0.00473 | 0.00116 |
| 1.99 | 1.4 | 0.51286 | -0.00216 | 0.81088 | -0.00093 | -0.00091 |

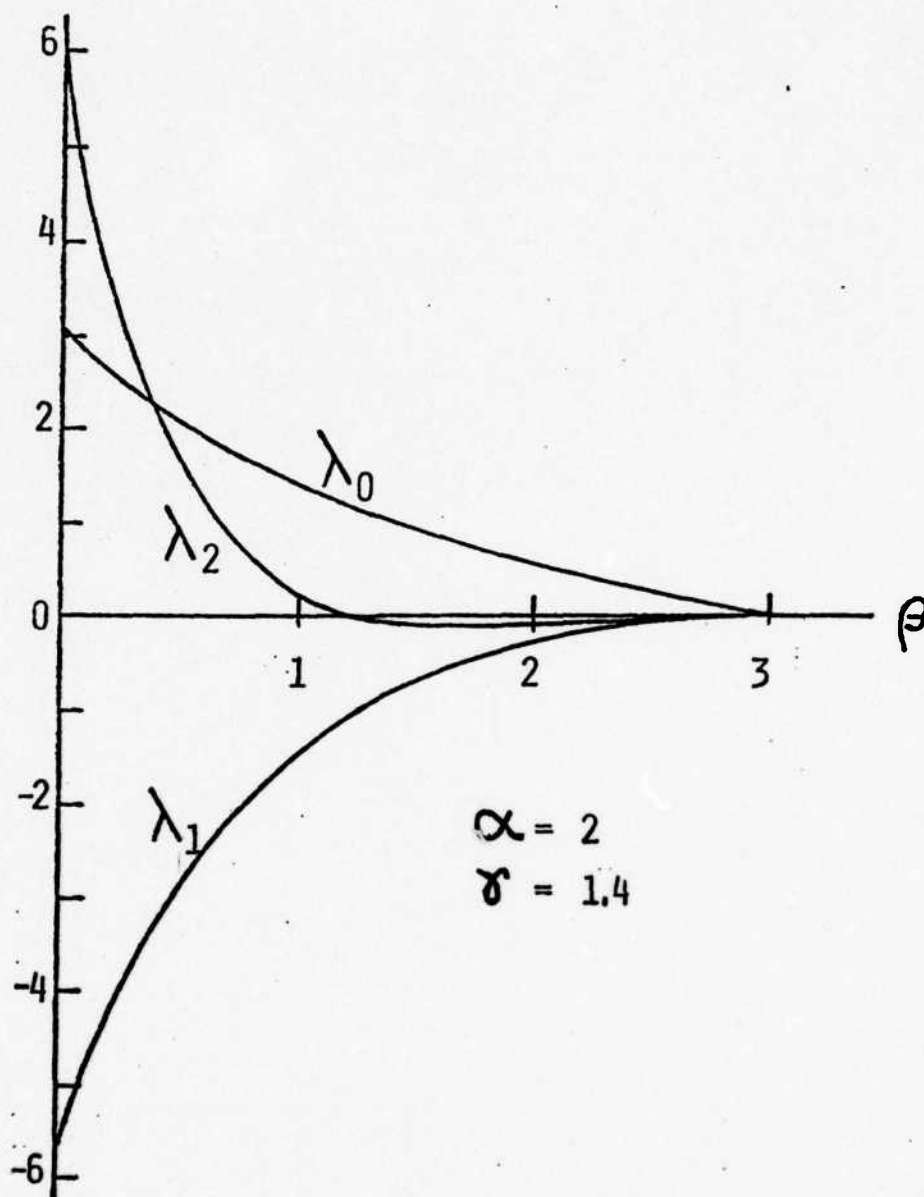
TABLE 1. EXPANSION COEFFICIENTS FOR CYLINDRICAL WAVE, $\alpha = 1$

| B | γ | J_0 | λ_1 | J_1 | λ_2 | J_2 |
|----------|---|---|--|--|---|---|
| 0 ↓ | 1.1 1.2 1.3 1.4 5/3 2 3 | 1.87082 1.02644 0.73942 0.59261 0.40915 0.31034 0.19774 | -6.00300 -5.89969 -5.81883 -5.75446 -5.63556 -5.54866 -5.44223 | -0.41019 -0.35189 -0.32307 -0.30338 -0.26860 -0.24066 -0.19204 | 7.75197 6.80141 6.15881 5.69937 4.94952 4.46598 3.91799 | 2.41710 1.16354 0.75899 0.56291 0.33752 0.23100 0.12912 |
| .25 | 1.4 | 0.49105 | -4.07792 | -0.03462 | 3.14452 | 0.36095 |
| .5 ↓ | 1.1 1.4 | 0.57496 0.44325 | -3.03230 -2.87521 | 2.35264 0.11646 | 2.33198 1.57621 | 0.43541 0.23932 |
| 1.0 ↓ | 1.1 1.2 1.3 1.4 5/3 | 0.44272 0.42440 0.40974 0.39761 0.37295 | -1.43972 -1.42411 -1.41519 -1.41027 -1.40748 | 2.72249 1.08744 0.55541 0.29596 -0.00305 | 0.34785 0.28346 0.24441 0.21973 0.18889 | 0.14331 0.12362 0.11135 0.10322 0.09144 |
| 1.5 ↓ | 1.1 1.4 | 0.39238 0.37556 | -0.66360 -0.66763 | 2.88715 0.40369 | -0.07706 -0.08871 | 0.04510 0.04054 |
| 2.0 ↓ | 1.1 1.2 1.3 1.4 5/3 | 0.36584 0.36489 0.36379 0.36257 0.35879 | -0.27807 -0.28116 -0.28403 -0.28669 -0.29290 | 2.98455 1.31493 0.75685 0.47695 0.13969 | -0.09847 -0.09717 -0.09595 -0.09485 -0.09243 | 0.00554 0.00764 0.00959 0.01137 0.01542 |
| 2.5 ↓ | 1.1 1.4 | 0.34945 0.35401 | -0.08812 -0.09310 | 3.05043 0.53054 | -0.04453 -0.04237 | -0.01159 -0.00205 |
| 2.95 | 1.4 | 0.34846 | -0.00629 | 0.56801 | -0.00320 | -0.00752 |
| 2.99 | 1.4 | 0.34804 | -0.00121 | 0.57097 | -0.00062 | -0.00782 |

TABLE 2. EXPANSION COEFFICIENTS FOR SPHERICAL WAVE, $\alpha = 2$

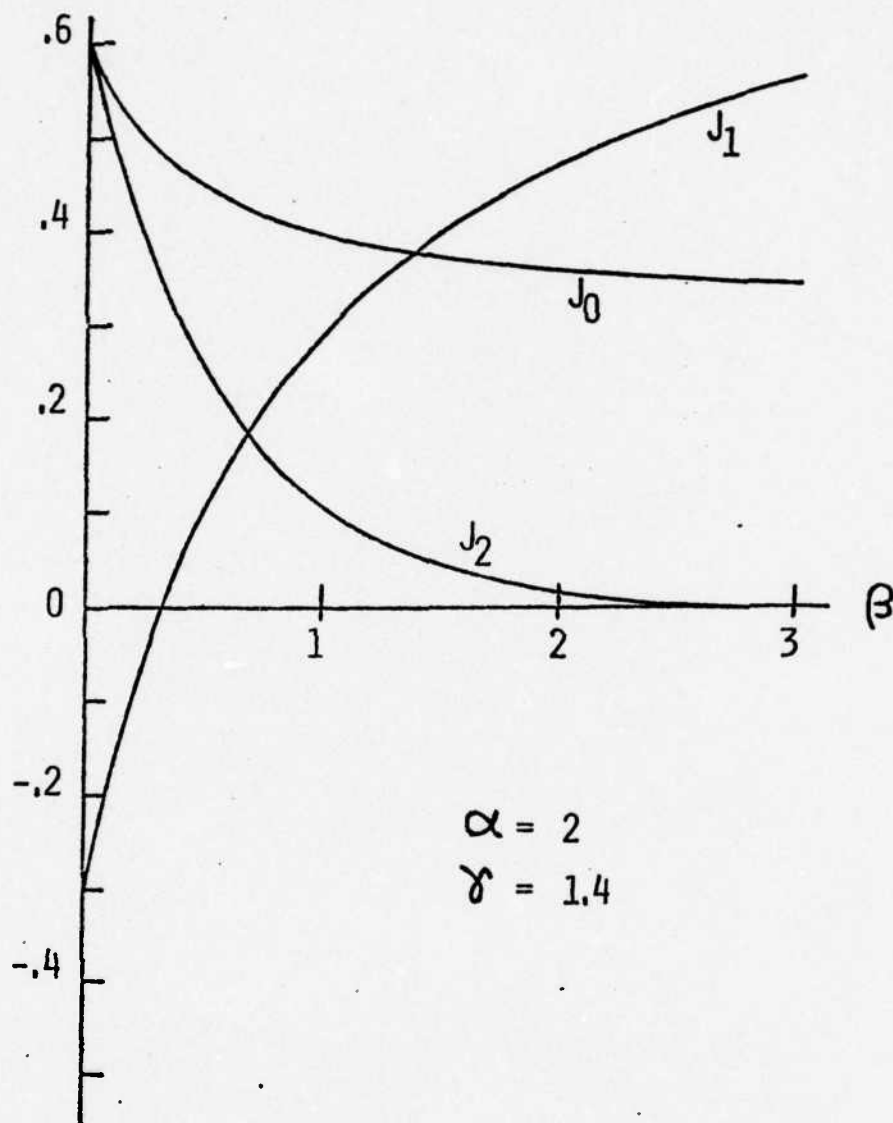
| β | γ | J_0 | λ_1 | J_1 | λ_2 | J_2 |
|---------|---|---|--|--|---|--|
| 0 | 1.1 1.2 1.3 1.4 5/3 2 3 | 5.57737 3.02287 2.14757 1.69705 1.13051 0.82425 0.47899 | -2.32573 -2.24368 -2.18618 -2.14334 -2.06835 -2.01427 -1.94071 | -2.97147 -1.78234 -1.36163 -1.13735 -0.83828 -0.66025 -0.42959 | 4.19515 3.45473 3.01386 2.72112 2.27262 1.99397 1.66803 | 11.69897 5.22159 3.23624 2.30893 1.28460 0.82176 0.39949 |
| .25 | 1.4 | 1.23367 | -1.20673 | 0.08085 | 1.19867 | 1.32576 |
| .5 ↓ | 1.1 1.4 | 1.25640 1.08643 | -0.57074 -0.56662 | 7.87118 0.67245 | 0.37492 0.34222 | 0.91993 0.74140 |
| .75 | 1.4 | 1.01463 | -0.19267 | 1.07922 | 0.04087 | 0.36683 |
| .95 | 1.4 | 0.97925 | -0.02768 | 1.33264 | -0.00185 | 0.17582 |
| .99 | 1.4 | 0.97358 | -0.00517 | 1.37783 | -0.00061 | 0.14599 |

TABLE 3. EXPANSION COEFFICIENTS FOR PLANE WAVE, $\alpha = 0$



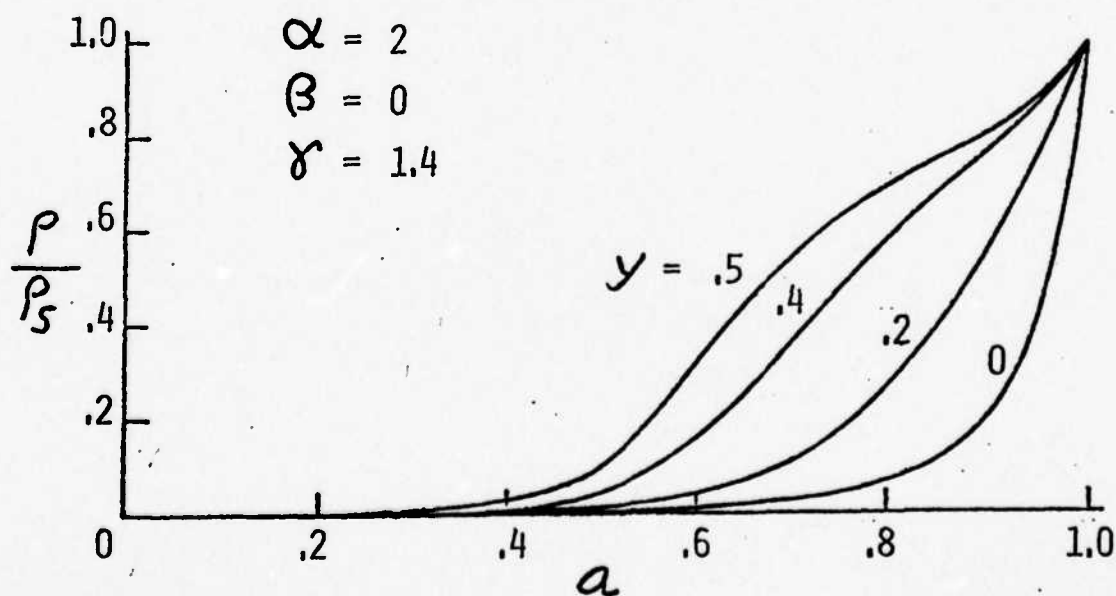
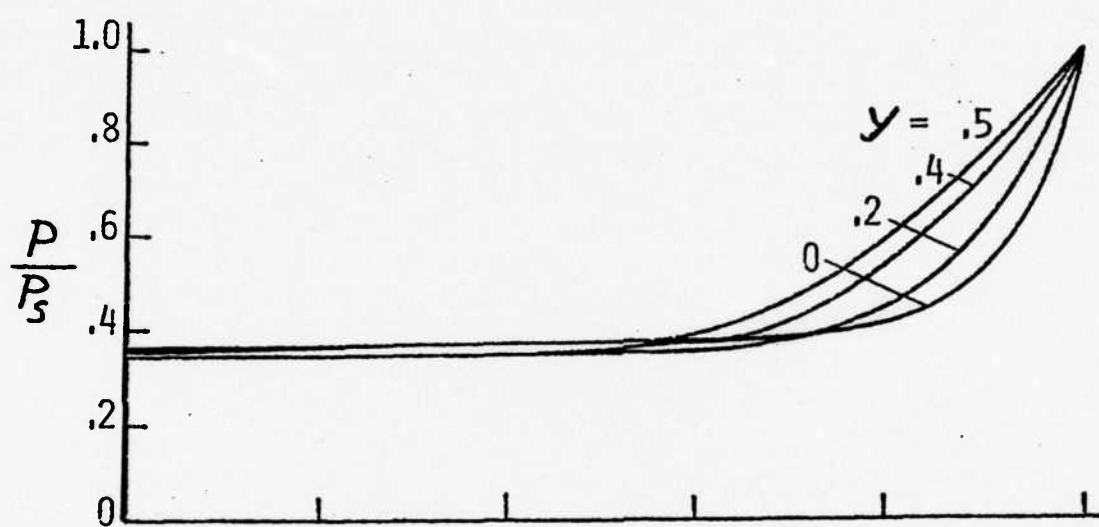
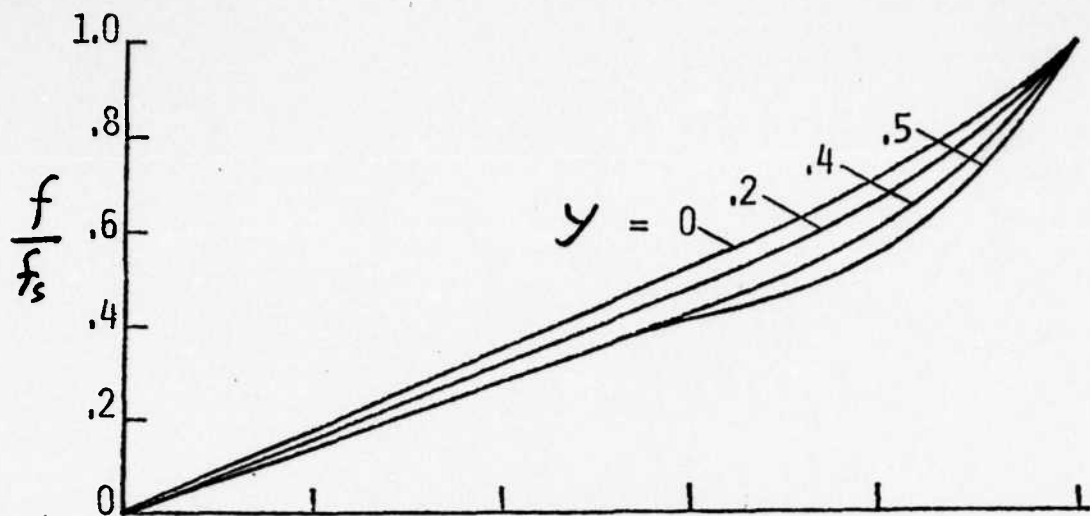
Coefficients for Decay Index Expansion as a Function of Energy Input Parameter.

Figure 1

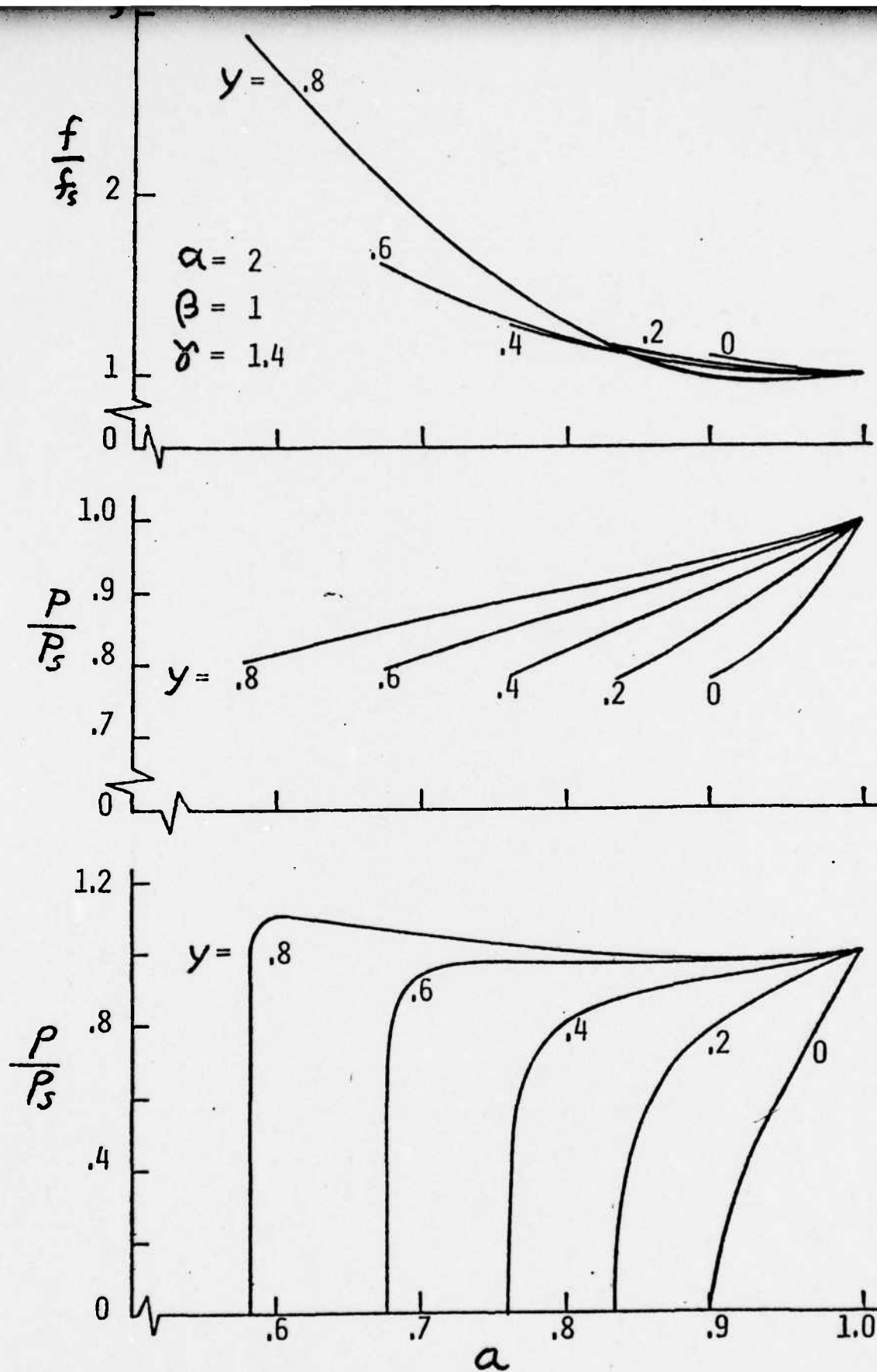


Coefficients of Energy Integral Expansion as a Function of Energy Input Parameter.

Figure 2

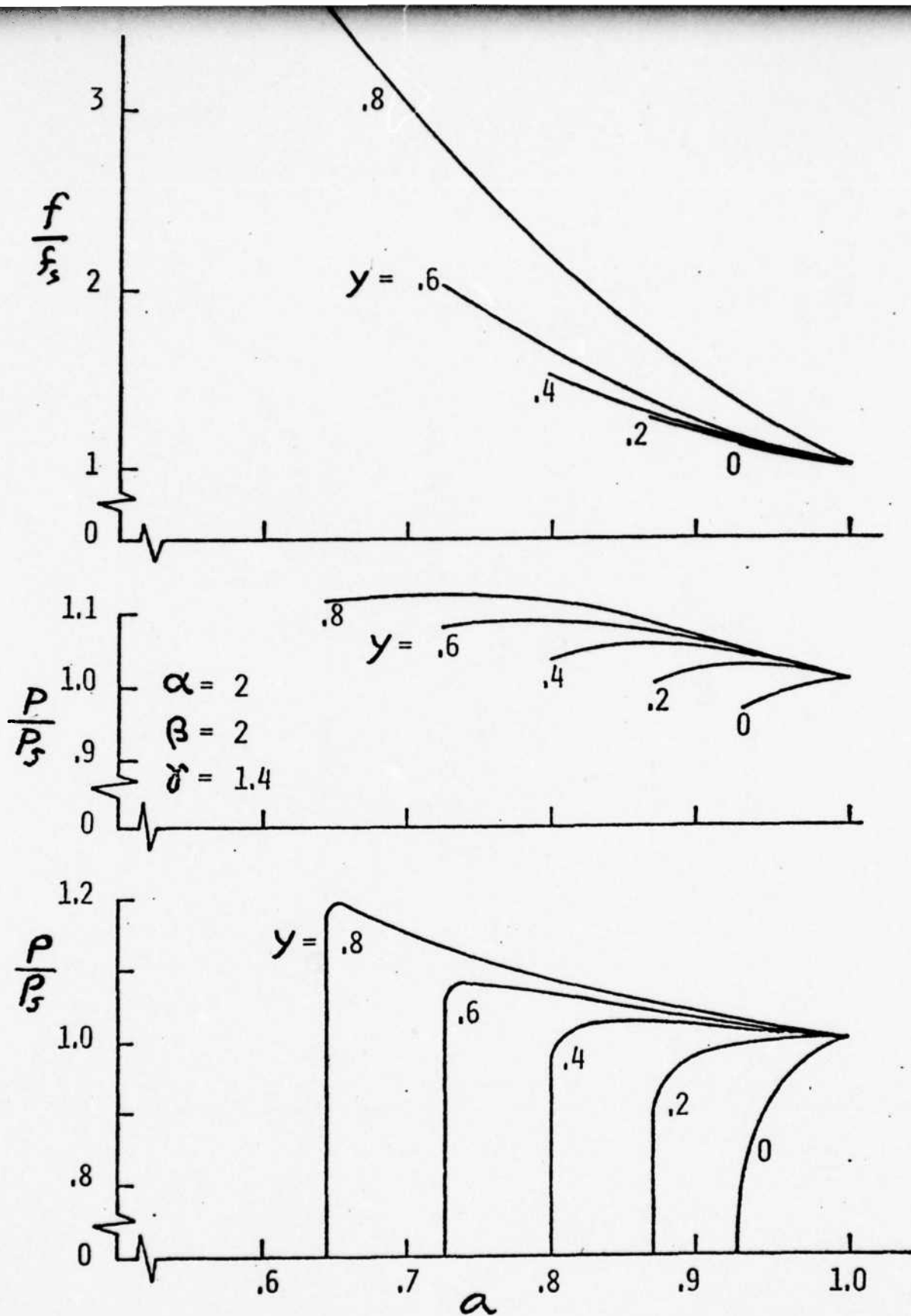


Flow Field Structure Behind Spherical Blast Wave for Instantaneous Energy Input.

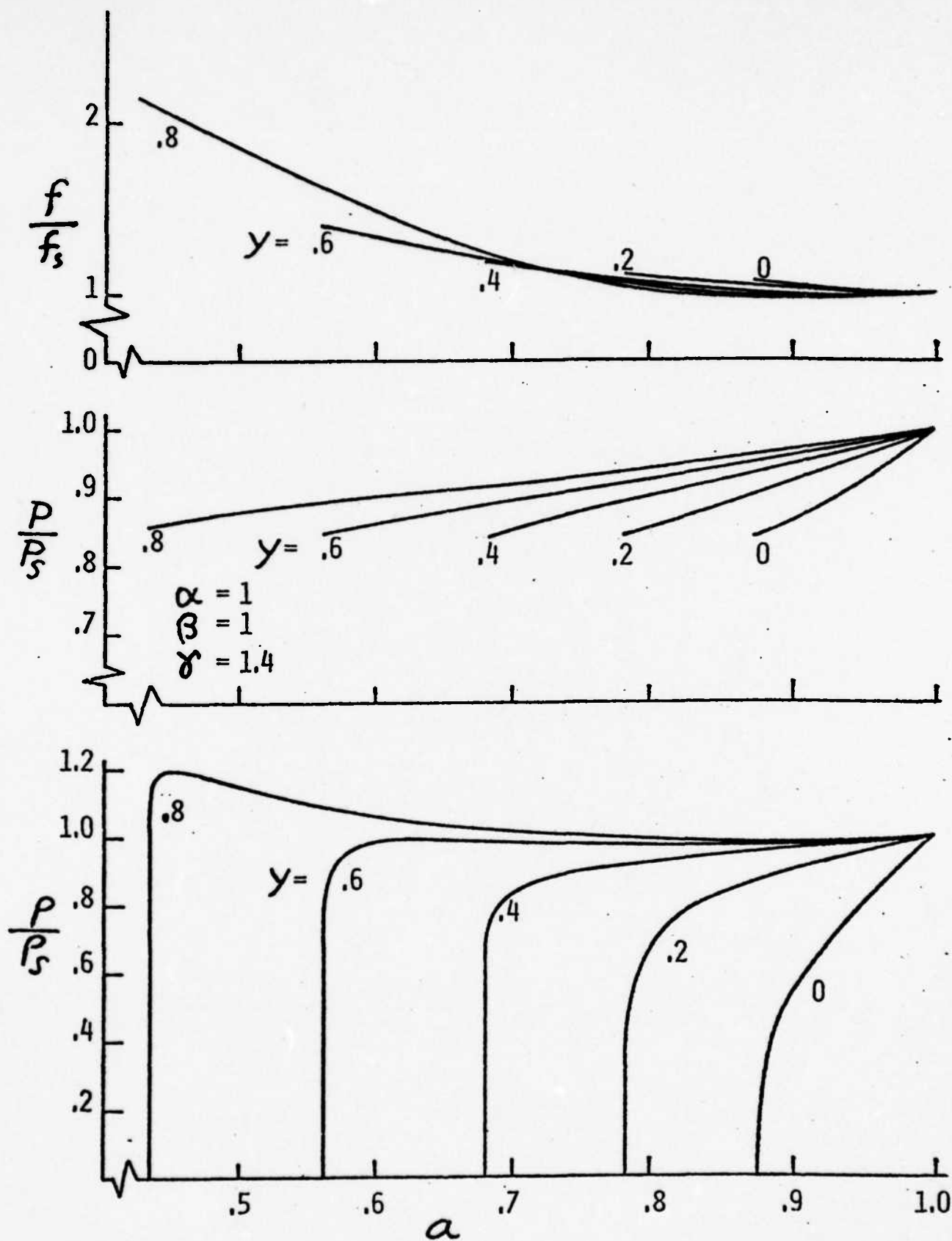


Flow Field Structure Behind Spherical Blast Wave for Constant Energy Input Rate.

Figure 4

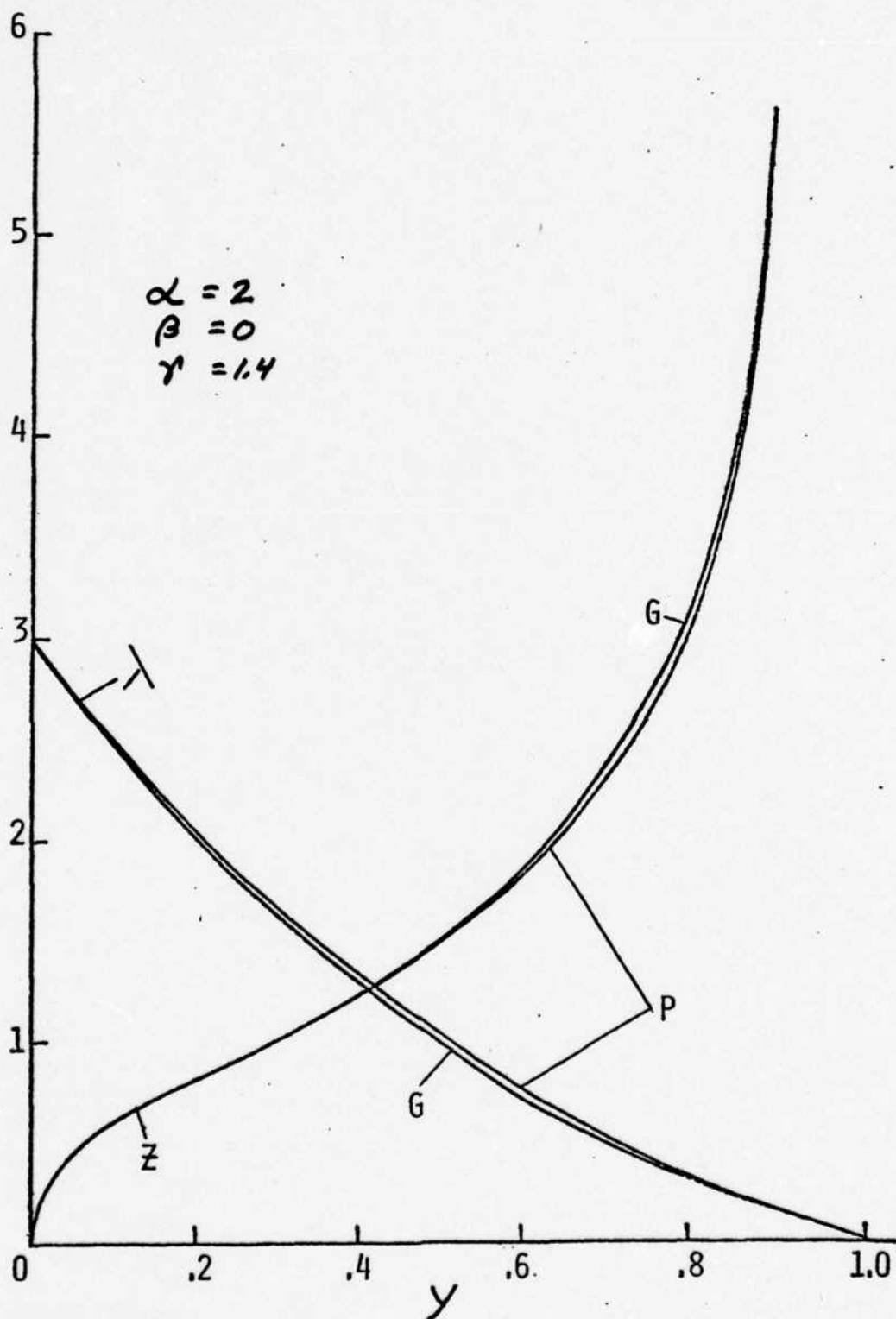


Flow Field Structure Behind Spherical Blast Wave for Linearly Increasing Energy Input Rate.

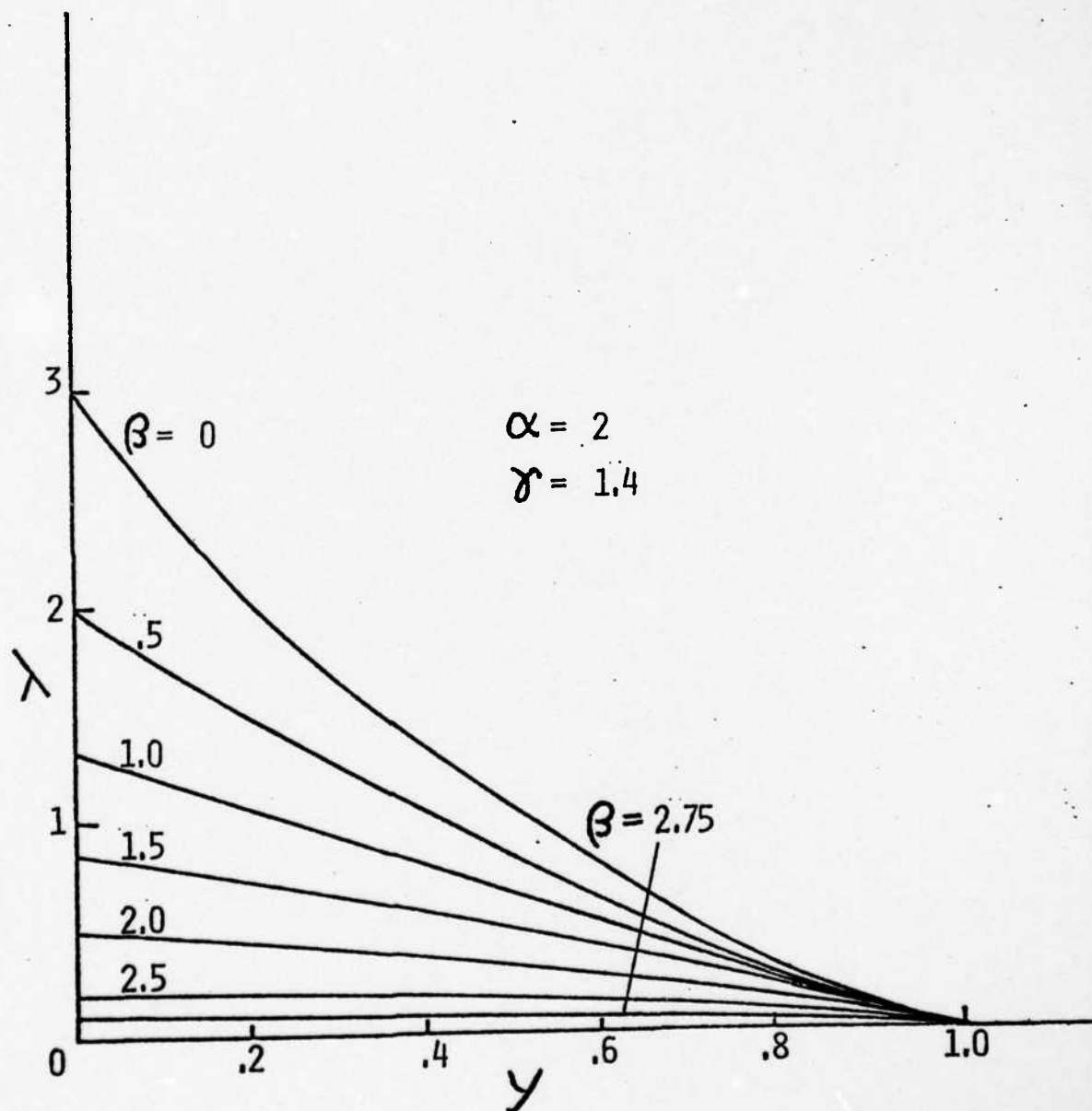


Flow Field Structure Behind Cylindrical Blast Wave for Constant Energy Input Rate.

Figure 6

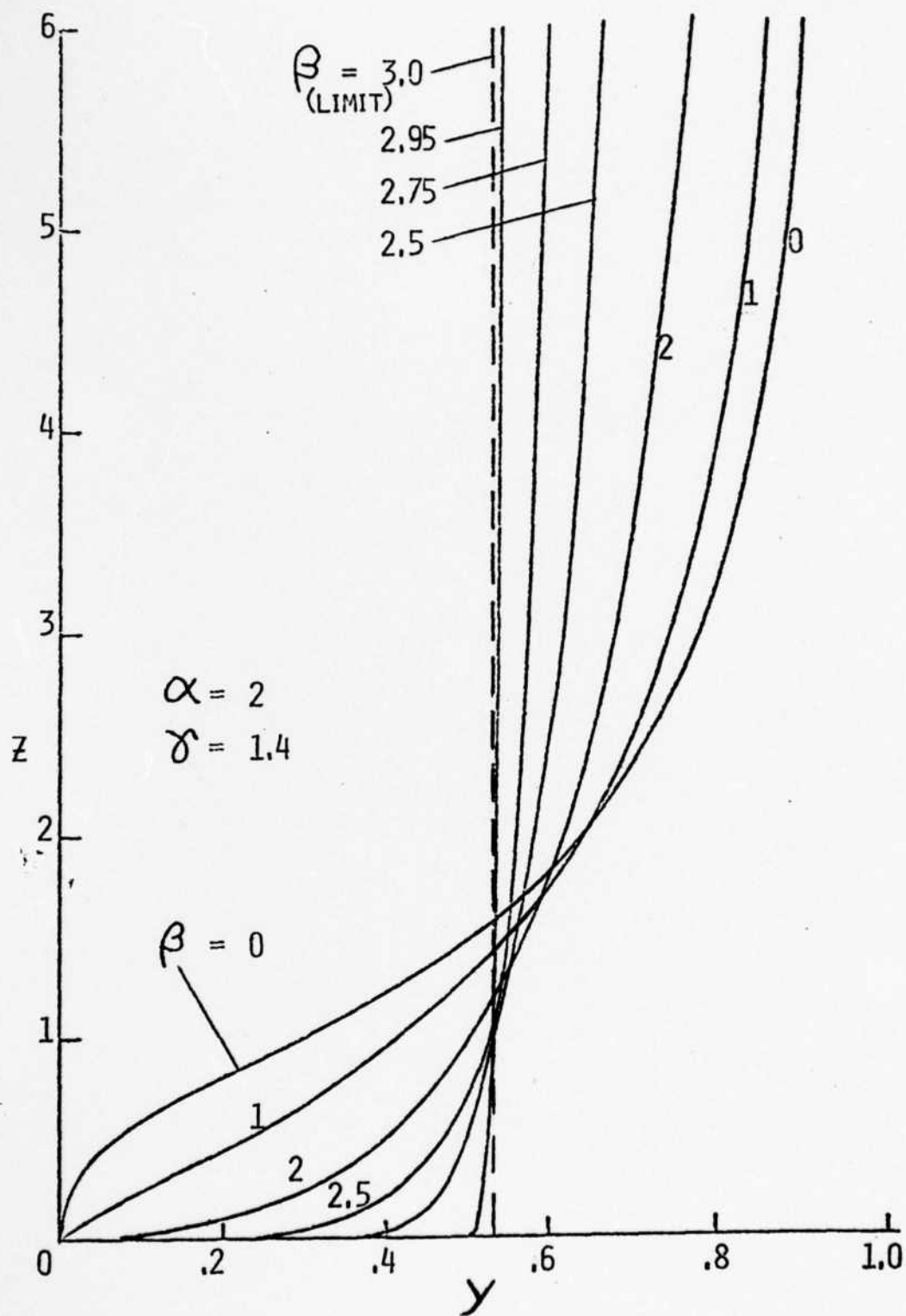


Decay Index and Shock Front Position from Second Order Perturbation Solution (P) Compared with "Exact Solution" of Goldstine and Von Neumann (G)



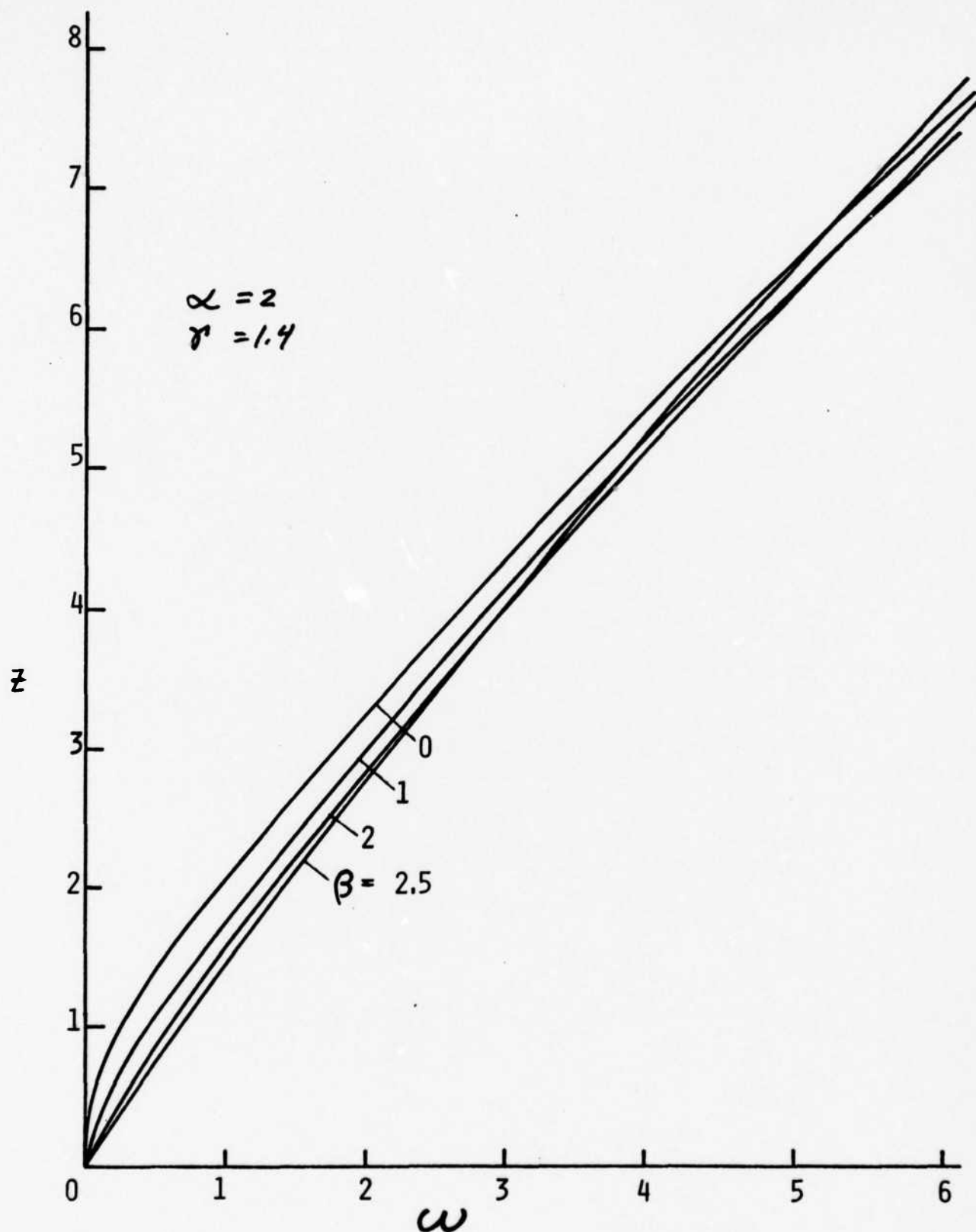
Variation of Decay Index with Shock Strength.

Figure 8



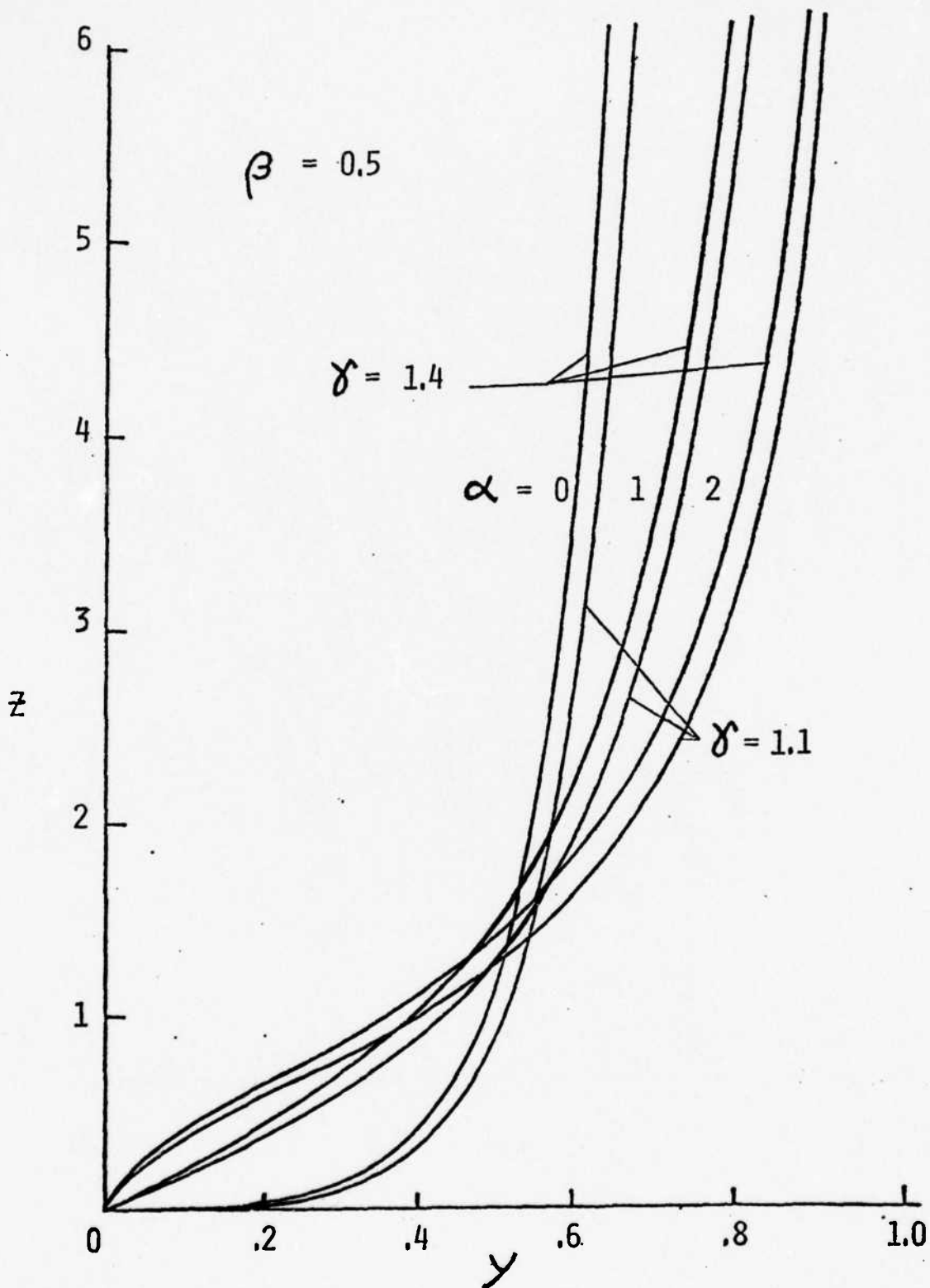
Shock Position as a Function of Shock Strength and Energy Input Parameter.

Figure 9



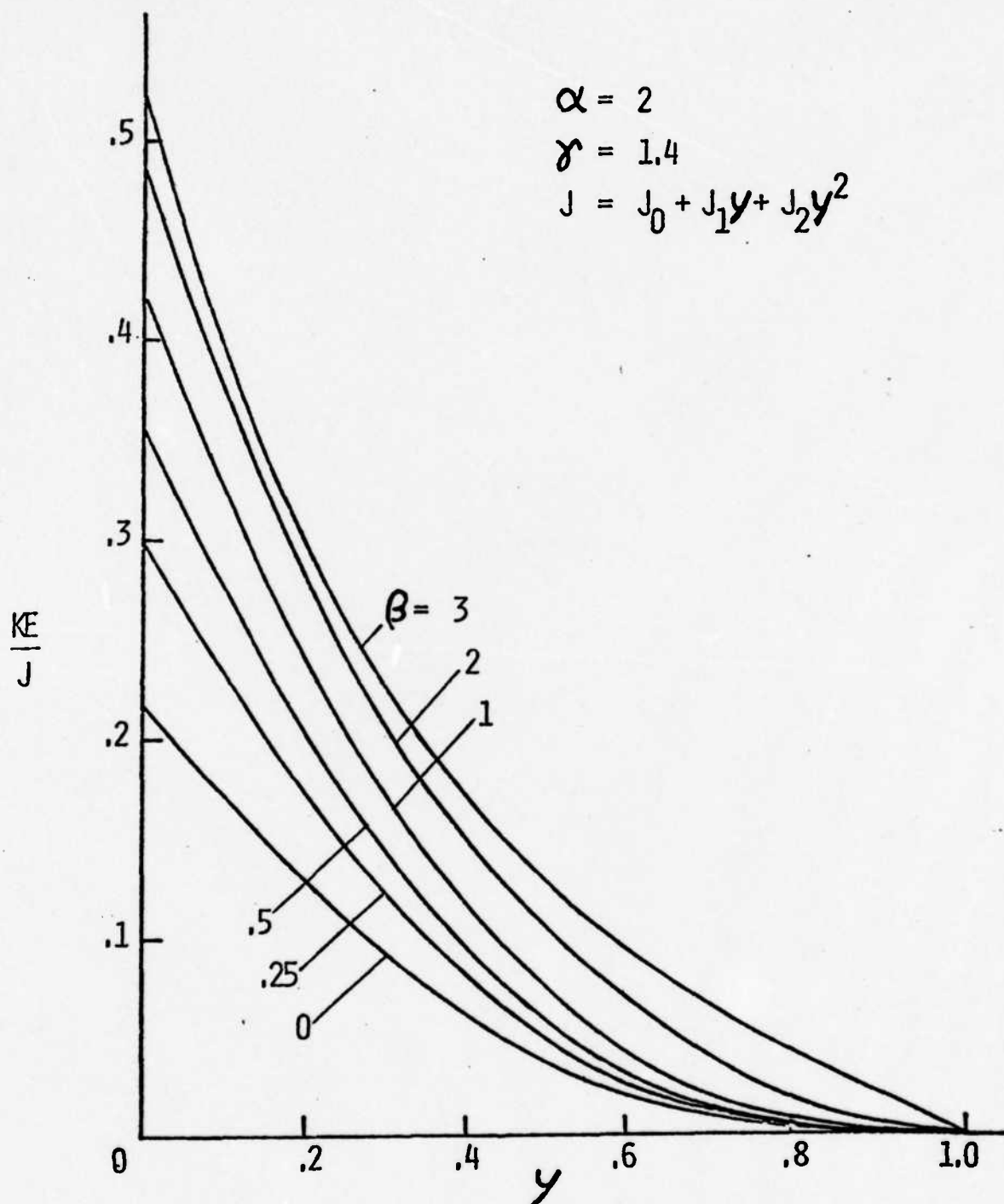
Shock Position as a Function of Time.

Figure 10



Shock Positions for Plane, Cylindrical and Spherical Blasts Compared.

Figure 11



Kinetic Energy Fraction Variation with Shock Strength and Energy Input Parameter.

Figure 12.

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APPENDIX A

Summary of first and second order equations required for blast wave computation

FIRST ORDER:

$$(A-1) \quad K_0 = \frac{2\gamma}{\gamma+1} \left[\frac{\gamma-1}{(\alpha+1)(\gamma+1)} \right]^\gamma$$

$$(A-2) \quad K_{11} = 2\gamma/(\gamma-1) - (\gamma-1)/2\gamma$$

$$(A-3) \quad e_2 = 1 + (\gamma-1)\lambda_0/\gamma(\alpha+1)$$

$$(A-4) \quad e_1 = 1 - \lambda_0/\gamma(\alpha+1)$$

$$(A-5) \quad e_3 = 1 + (2\gamma-1)\lambda_0/\gamma(\alpha+1)$$

$$(A-6) \quad g_0 = K_0/(a_0^\alpha a_0') \gamma m \lambda_0/(\alpha+1)$$

$$(A-7) \quad A_0 = \gamma(\alpha+1)[a_0^\alpha g_0/a_0' - (\alpha+1)m^2]$$

$$(A-8) \quad B_0 = \gamma(\alpha+1)(2\alpha+\lambda_0)m/2$$

$$(A-9) \quad C_0 = \gamma[\alpha(\alpha+1)g_0 a_0^\alpha a_0'/a_0^2 + \lambda_0/2]$$

$$(A-10) \quad F = \gamma(a_0 - (\alpha+1)m a_0')$$

$$(A-11) \quad B_1 = B_0 + \gamma(\alpha+1) \left[a_0^\alpha \frac{g_0 a_0''}{(a_0')^2} - \frac{g_0'}{a_0'} - \frac{g_0}{a_0} - 2m\lambda_0 \right]$$

$$(A-12) \quad C_1 = C_0 + \gamma\lambda_0^2/2 - \alpha(\alpha+1)(\gamma-1)a_0^\alpha g_0'/a_0$$

$$(A-13) \quad D_1 = K_{11} a_0^\alpha m \lambda_0/(\alpha+1) [(\alpha+1)g_0' + g_0 \lambda_0/m]$$

$$(A-14) \quad E_1 = [(\alpha+1)a_0^\alpha g_0' - D_1/K_{11}]/\lambda_0 - F/2$$

$$(A-15) \quad a_{11}'' = (B_1 a_{11}' + C_1 a_{11} + D_1)/A_0$$

$$(A-16) \quad a_{12}'' = (B_1 a_{12}' + C_1 a_{12} + E_1)/A_0$$

$$(A-17) \quad J_{11}' = \frac{F}{(\alpha+1)} [(1+\lambda_0) a_{11} - (\alpha+1) m a_{11}'] \\ + g_0 a_0^\alpha a_0' \left[K_{11} \frac{m \lambda_0 / (\alpha+1)}{(\gamma-1)} - \left(\frac{a_{11}'}{a_0'} + \alpha \frac{a_{11}}{a_0} \right) \right]$$

$$(A-18) \quad J_{12}' = \frac{F}{(\alpha+1)} [(1+\lambda_0) a_{12} - (\alpha+1) m a_{12}'] \\ + g_0 a_0^\alpha a_0' \left[\frac{1-m \lambda_0 / (\alpha+1)}{(\gamma-1) \lambda_0} - \left(\frac{a_{12}'}{a_0'} + \alpha \frac{a_{12}}{a_0} \right) \right]$$

$$(A-19) \quad \frac{g_{11}}{g_0} = K_{11} m \lambda_0 / (\alpha+1) - \gamma \left(\frac{a_{11}'}{a_0'} + \alpha \frac{a_{11}}{a_0} \right)$$

$$(A-20) \quad \frac{g_{12}}{g_0} = \frac{1-m \lambda_0 / (\alpha+1)}{\lambda_0} - \gamma \left(\frac{a_{12}'}{a_0'} + \alpha \frac{a_{12}}{a_0} \right)$$

$$(A-21) \quad \Delta J_{11} = \int_0^\delta J_{11}' dm = \frac{F\delta}{(\alpha+1)} [(1+\lambda_0) a_{11} - (\alpha+1) \delta a_{11}']_\delta \\ + \left[g_0 \left(\frac{K_0}{g_0} \right)^{1/\gamma} \right]_\delta \left[K_{11} \frac{\delta e_2}{e_2 (\gamma-1)} - \left(\frac{a_{11}'}{a_0'} + \alpha \frac{a_{11}}{a_0} \right) \frac{\delta e_1}{e_1} \right]$$

$$(A-22) \quad \Delta J_{12} = \int_0^\delta J_{12}' dm = \frac{F\delta}{(\alpha+1)} [(1+\lambda_0) a_{12} - (\alpha+1) \delta a_{12}']_\delta \\ - \left[g_0 \left(\frac{K_0}{g_0} \right)^{1/\gamma} \right]_\delta \left(\frac{\delta e_2}{e_2 \lambda_0 (\gamma-1)} + \left[\frac{a_{12}'}{a_0'} + \alpha \frac{a_{12}}{a_0} - \frac{1}{\lambda_0 (\gamma-1)} \right]_\delta \frac{\delta e_1}{e_1} \right)$$

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SECOND ORDER:

$$(A-23) \quad K_1 = K_{11} - \lambda_1/\lambda_0$$

$$(A-24) \quad K_{21} = K_{11} \lambda_1/\lambda_0 - 2\gamma/(\gamma-1)^2 - (\gamma-1)^2/8\gamma^2 - \lambda_1^2/2\lambda_0^2$$

$$(A-25) \quad a_1 = a_{11} + \lambda_1 a_{12}$$

$$(A-26) \quad g_1/g_0 = g_{11}/g_0 + \lambda_1 g_{12}/g_0$$

$$(A-27) \quad \frac{g_1'}{g_0} = \frac{g_1 g_0'}{g_0^2} + \frac{K_{21} \lambda_0 m^{\lambda_0/(\alpha+1)}}{(\alpha+1)m}$$

$$- \gamma \left[\alpha \left(\frac{a_1'}{a_0} - \frac{a_1 a_0'}{a_0^2} \right) + \frac{a_1''}{a_0'} - \frac{a_1' a_0''}{(a_0')^2} \right]$$

$$(A-28) \quad G = a_1(1+\lambda_0) - (\alpha+1)ma_1'$$

$$(A-29) \quad B_2 = B_1 - 2(\alpha+1)\gamma m \lambda_0$$

$$(A-30) \quad C_2 = C_1 + (1+5\lambda_0/2)\gamma \lambda_0$$

$$(A-31) \quad D_2 = (\alpha+1)a_0^\alpha \left\{ K_{21} \lambda_0 \left[\frac{g_0'}{\lambda_0} + \frac{2g_0}{(\alpha+1)m} \right] m^{2\lambda_0/(\alpha+1)} \right.$$

$$- K_{11} \lambda_1 \left[\frac{g_0'}{\lambda_0} + \frac{g_0}{(\alpha+1)m} \right] m^{\lambda_0/(\alpha+1)} + g_1' \left[\frac{g_1}{g_0} + \frac{a_1}{a_0} \right]$$

$$+ \frac{g_0'}{2} \left(\gamma \left[\alpha \frac{a_1^2}{a_0^2} + \left(\frac{a_1'}{a_0'} \right)^2 \right] - \frac{g_1^2}{g_0^2} - \frac{\lambda_1^2}{\lambda_0^2} \right)$$

$$+ \gamma g_0 \left(\frac{\alpha a_1}{a_0^3} [a_0 a_1' - a_1 a_0'] + \frac{1}{(a_0')^3} [a_0' a_1' a_1'' - (a_1')^2 a_0''] \right) \Bigg\}$$

$$+ \frac{\gamma \lambda_1}{2} [a_1(1+\lambda_0) - 3(\alpha+1)ma_1'] + \frac{1}{2}(\alpha-1)\alpha(\alpha+1)a_1^2 g_0'$$

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$$(A-32) \quad E_2 = a_0^\alpha \left(\frac{(\alpha+1)g_0'}{2\lambda_0} \left[1 - m^{2\lambda_0/(\alpha+1)} \right] - \frac{g_0}{m} m^{2\lambda_0/(\alpha+1)} \right) - \frac{F}{2}$$

$$(A-33) \quad a_{21}'' = (B_2 a_{21}' + C_2 a_{21} + D_2) / A_0$$

$$(A-34) \quad a_{22}'' = (B_2 a_{22}' + C_2 a_{22} + E_2) / A_0$$

$$(A-35) \quad J_{21}' = \frac{1}{\alpha+1} [F(\lambda_1 a_1 + (1+2\lambda_0) a_{21} - (\alpha+1) m a_{21}') + \frac{\gamma}{2} G^2] \\ + \frac{g_0 a_0^\alpha a_0'}{\gamma-1} \left[\frac{a_{21}'}{a_0'} + \frac{\alpha a_{21}}{a_0} + \frac{\alpha a_1}{a_0} \left(\frac{a_1'}{a_0'} + \frac{(\alpha-1) a_1}{2 a_0} \right) + \frac{g_{21}}{g_0} \right. \\ \left. + \frac{g_1}{g_0} \left(\frac{a_1'}{a_0'} + \frac{\alpha a_1}{a_0} \right) \right]$$

$$(A-36) \quad J_{22}' = \frac{F}{\alpha+1} [(1+2\lambda_0) a_{22} - (\alpha+1) m a_{22}'] \\ + \frac{g_0 a_0^\alpha a_0'}{\gamma-1} \left[\frac{a_{22}'}{a_0'} + \frac{\alpha a_{22}}{a_0} + \frac{g_{22}}{g_0} \right]$$

$$(A-37) \quad \frac{g_{21}}{g_0} = \frac{1}{2} \left(\frac{g_1^2}{g_0^2} - \frac{\lambda_1^2}{\lambda_0^2} \right) - \gamma \left[\alpha \left(\frac{a_{21}}{a_0} - \frac{a_1^2}{2 a_0^2} \right) + \frac{a_{21}'}{a_0'} - \frac{1}{2} \left(\frac{a_1'}{a_0'} \right)^2 \right] \\ + K_{21} m^{2\lambda_0/(\alpha+1)} - \frac{K_1 \lambda_1 m^{\lambda_0/(\alpha+1)}}{\lambda_0}$$

$$(A-38) \quad \frac{g_{22}}{g_0} = \frac{1}{2\lambda_0} \left(1 - m^{2\lambda_0/(\alpha+1)} \right) - \gamma \left(\frac{\alpha a_{22}}{a_0} + \frac{a_{22}'}{a_0'} \right)$$

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$$(A-39) \quad \Delta J_{21} = \int_0^\delta J'_{21} dm = \frac{\delta}{\alpha+1} \left[F(\lambda_1 a_1 + (1+2\lambda_0) a_{21} - (\alpha+1) \delta a'_{21} + \frac{\gamma}{2} G^2) \right]_\delta$$

$$+ \left[\frac{g_0}{\gamma-1} \left(\frac{K_0}{g_0} \right)^{1/\gamma} \right]_\delta \left\{ \frac{\delta^{e_1}}{e_1} \left[\frac{g_1}{g_0} \left(\frac{a'_1}{a_0} + \frac{\alpha a_1}{a_0} \right) + \frac{\alpha a_1}{a_0} \left(\frac{a'_1}{a_0} + \frac{(\alpha-1)a_1}{2a_0} \right) \right. \right. \\ \left. \left. + \frac{1}{2} \left[\frac{g_1^2}{g_0^2} - \frac{\lambda_1^2}{\lambda_0^2} + \gamma \left(\frac{\alpha a_1^2}{a_0^2} + \left(\frac{a'_1}{a_0} \right)^2 \right) \right] - (\gamma-1) \left(\frac{\alpha a_{21}}{a_0} + \frac{a'_{21}}{a_0} \right) \right] \right\} \\ + \frac{K_{21} \delta^{e_3}}{e_3} - \frac{K_1 \lambda_1}{\lambda_0} \frac{\delta^{e_2}}{e_2} \Bigg\}$$

$$(A-40) \quad \Delta J_{22} = \int_0^\delta J'_{22} dm = \frac{\delta}{\alpha+1} \left[F(a_{22}(1+2\lambda_0) - (\alpha+1) \delta a'_{22}) \right]_\delta$$

$$+ \left[\frac{g_0}{\gamma-1} \left(\frac{K_0}{g_0} \right)^{1/\gamma} \right]_\delta \left\{ \frac{1}{2\lambda_0} \left(\frac{\delta^{e_1}}{e_1} - \frac{\delta^{e_3}}{e_3} \right) - (\gamma-1) \left(\frac{\alpha a_{22}}{a_0} + \frac{a'_{22}}{a_0} \right) \frac{\delta^{e_1}}{e_1} \right\}$$

APPENDIX B - COMPUTER PROGRAMS

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| | |
|---|----------|
| 7A12(J) = 0.00 | BLST 510 |
| ZA21(J) = 0.00 | BLST 520 |
| ZA22(J) = 0.00 | BLST 530 |
| ZAP11(J) = Z(5) | BLST 540 |
| ZAP12(J) = 0.00 | BLST 550 |
| ZAP21(J) = 0.00 | BLST 560 |
| ZAP22(J) = 0.00 | BLST 570 |
| ZG(J) = KN/Z(2)**GAMMA | BLST 580 |
| ZG11(J) = ZG(J)*(AK11 - GAMMA*Z(5)/Z(2)) | BLST 590 |
| ZG12(J) = 0.00 | BLST 600 |
| ZG21(J) = 0.00 | BLST 610 |
| ZG22(J) = 0.00 | BLST 620 |
| 25 WRITE(6,42) ZA(J),ZAP(J),Z(3),ZG(J),ZM(J) | BLST 630 |
| IF(KPASS.EQ.2) GO TO 30 | BLST 640 |
| WRITE(6,43) ZA11(J),ZAP11(J),ZA12(J),ZAP12(J),Z(8),Z(9) | BLST 650 |
| IF(KPASS.EQ.1) GO TO 50 | BLST 660 |
| 30 WRITE(6,44) ZA21(J),ZAP21(J),ZA22(J),ZAP22(J),Z(14),Z(15) | BLST 670 |
| 40 FORMAT(////) | BLST 680 |
| 41 FORMAT(' ALFA =',F5.3,3X,'BETA =',F5.3,3X,'GAMMA =',F6.4,/)) | BLST 690 |
| 42 FORMAT(/,3X,'AO =',E10.4,3X,'AOPRIME =',E10.4,3X,'JO =',E11.5,3X | BLST 700 |
| 1,'GO =',E11.5,3X,'MASS =',F10.3) | BLST 710 |
| 43 FORMAT(/,3X,'A11=',E10.4,3X,'A11PRIME=',E10.4,3X,'A12=',E10.4,3X | BLST 720 |
| 1,'A12PRIME=',E10.4,3X,'J11=',E11.5,3X,'J12=',E11.5,/)) | BLST 730 |
| 44 FORMAT(/,3X,'A21=',E10.4,3X,'A21PRIME=',E10.4,3X,'A22=',E10.4,3X | BLST 740 |
| 1,'A22PRIME=',E10.4,3X,'J21=',E11.5,3X,'J22=',E11.5,/)) | BLST 750 |
| 45 FORMAT(/,' DJO =',F10.5,2X,'EKO =',F10.5,2X,'EFRACO =',F10.5) | BLST 760 |
| 46 FORMAT(/,' PLAM =',F10.5,2X,'K21 =',F10.5) | BLST 770 |
| 47 FORMAT(/,' DJ11 =',F10.5,2X,'DJ12 =',F10.5,2X,'LAM1 =',F10.5,2X, | BLST 780 |
| 1'J1 =',F10.5,2X,'EK1 =',F10.5,2X,'K1 =',F10.5) | BLST 790 |
| 48 FORMAT(/,' DJ21 =',F10.5,2X,'DJ22 =',F10.5,2X,'LAM2 =',F10.5,2X, | BLST 800 |
| 1'J2 =',F10.5,2X,'EK2 =',F10.5) | BLST 810 |
| COMMENCE INTEGRATION | BLST 820 |
| 50 DO 100 I=1,KK | BLST 830 |
| 100 CALL RUNKUT(Z,MASS,C,STEP,NV) | BLST 840 |
| J = J-1 | BLST 850 |
| FM1 = MASS**(LAMN/ALP) | BLST 860 |
| IF(KPASS.EQ.2) GO TO 98 | BLST 870 |
| ZM(J) = MASS | BLST 880 |
| ZA(J) = Z(1) | BLST 890 |
| ZAP(J) = Z(2) | BLST 900 |
| ZG(J) = KN/((Z(1)**L*Z(2))**GAMMA*FM1) | BLST 910 |
| ZA11(J) = Z(4) | BLST 920 |
| ZA12(J) = Z(6) | BLST 930 |
| ZAP11(J) = Z(5) | BLST 940 |
| ZAP12(J) = Z(7) | BLST 950 |
| ZG11(J) = ZG(J)*(AK11*FM1 - GAMMA*(ALFA*Z(4)/Z(1)+Z(5)/Z(2))) | BLST 960 |
| ZG12(J) = ZG(J)*((1.00-FM1)/LAMN-GAMMA*(ALFA*Z(6)/Z(1)+Z(7)/Z(2))) | BLST 970 |
| 98 IF(KPASS.EQ.1) GO TO 99 | BLST 980 |
| ZA21(J) = Z(10) | BLST 990 |
| ZA22(J) = Z(12) | BLST1000 |
| ZAP21(J) = Z(11) | BLST1010 |
| ZAP22(J) = Z(13) | BLST1020 |
| ZG1 = ZG11(J) + LAM1*ZG12(J) | BLST1030 |
| ZA1 = ZA11(J) + LAM1*ZA12(J) | BLST1040 |

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|---|----------|
| ZAP1 = Z(5) + LAM1*Z(7) | BLST1050 |
| ZG21(J) = (((ZG1/ZG(J))**2-RLAM**2)/2.DO - GAMMA*(ALFA*Z(10)/Z(1) | BLST1060 |
| 1 + Z(11)/Z(2) - (ALFA*(ZA1/Z(1))**2 + (ZAP1/Z(2))**2)/2.DO) | BLST1070 |
| 2 + AK21*FM1**2 - K1*RLAM*FM1)*ZG(J) | BLST1080 |
| ZG22(J) = ((1.DO-FM1**2)/2.DO/LAMN - GAMMA*(ALFA*Z(12)/Z(1) + | BLST1090 |
| 1 Z(13)/Z(2)))*ZG(J) | BLST1100 |
| CUT DOWN STEP SIZE AS MASS APPROACHES ZERO | BLST1110 |
| 99 IF(J.EQ.101) STEP = STEP/10.DO | BLST1120 |
| IF(J.EQ.101) GO TO 101 | BLST1130 |
| IF(J.EQ.11) GO TO 101 | BLST1140 |
| IF(J.GT.2) GO TO 50 | BLST1150 |
| 101 Z3 = -Z(3) | BLST1160 |
| WRITE(6,42) ZA(J),ZAP(J),Z3,ZG(J),ZM(J) | BLST1170 |
| IF(KPASS.EQ.2) GO TO 104 | BLST1180 |
| Z8 = -Z(8) | BLST1190 |
| Z9 = -Z(9) | BLST1200 |
| WRITE(6,43) ZA11(J),ZAP11(J),ZA12(J),ZAP12(J),Z8,Z9 | BLST1210 |
| IF(KPASS.EQ.1) GO TO 105 | BLST1220 |
| 104 Z14 = -Z(14) | BLST1230 |
| Z15 = -Z(15) | BLST1240 |
| WRITE(6,44) ZA21(J),ZAP21(J),ZA22(J),ZAP22(J),Z14,Z15 | BLST1250 |
| 105 IF(J.GT.2) GO TO 50 | BLST1260 |
| COMPUTE EXTRAPOLATED VALUES FOR MASS = 0 WITH GO = CONSTANT | BLST1270 |
| ENSTP = -.0000100 | BLST1280 |
| DO 202 JK=1,2 | BLST1290 |
| DO 200 I=1,90 | BLST1300 |
| 200 CALL RUNKUT(Z,MASS,C,ENSTP,NV) | BLST1310 |
| FM1 = MASS**((LAMN/ALP) | BLST1320 |
| GN = KN/((Z(1)**L*Z(2))**GAMMA*FM1) | BLST1330 |
| Z3 = -Z(3) | BLST1340 |
| WRITE(6,42) Z(1),Z(2),Z3,GN,MASS | BLST1350 |
| IF(KPASS.EQ.2) GO TO 199 | BLST1360 |
| Z8 = -Z(8) | BLST1370 |
| Z9 = -Z(9) | BLST1380 |
| WRITE(6,43) Z(4),Z(5),Z(6),Z(7),Z8,Z9 | BLST1390 |
| IF(KPASS.EQ.1) GO TO 201 | BLST1400 |
| 199 Z14 = -Z(14) | BLST1410 |
| Z15 = -Z(15) | BLST1420 |
| WRITE(6,44) Z(10),Z(11),Z(12),Z(13),Z14,Z15 | BLST1430 |
| 201 IF(JK.EQ.2) GO TO 202 | BLST1440 |
| Z25 = Z(2) | BLST1450 |
| Z45 = Z(4) | BLST1460 |
| Z55 = Z(5) | BLST1470 |
| Z65 = Z(6) | BLST1480 |
| Z75 = Z(7) | BLST1490 |
| GNS = GN | BLST1500 |
| ENSTP = ENSTP/10.DO | BLST1510 |
| IF(KPASS.EQ.1) GO TO 202 | BLST1520 |
| Z105 = Z(10) | BLST1530 |
| Z115 = Z(11) | BLST1540 |
| Z125 = Z(12) | BLST1550 |
| Z135 = Z(13) | BLST1560 |
| POP CONTINUE | BLST1570 |
| IF(KPASS.EQ.2) GO TO 206 | BLST1580 |

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| <pre> AN = Z(1) ANL = AN**L ANP = Z(2) FAJ = AN - ALP*ANP*MASS GNO = (GN/GNS)*GN GN = .7D0*GN + .3D0*GNO GNKF = GN*(KN/GN)**(1.D0/GAMMA)/GM FJ1 = GNKF*MASS**E1/E1 FJ2 = GNKF*MASS**E2/E2 FJ3 = GNKF*MASS**E3/E3 ANO = AN**LP - ALP*(GM/GN)*FJ1 IF(ANO.LE.1.D-13) ANO=1.D-13 ANO = ANO**(1.D0/ALP) ANPO = (Z(2)/Z2S)*Z(2) DJO = GAMMA*MASS*FAJ**2/ALP/2.D0 + FJ1 AJO = DJO - Z(3) EKO = DJO - FJ1 - Z(10) EFRACO = EKO/AJO AMASS = 0.D0 WRITE(6,42) ANO,ANPO,AJO,GNO,AMASS WRITE(6,45) DJO,EKO,EFRACO A11 = Z(4) AP11 = Z(5) A12 = Z(6) AP12 = Z(7) A110 = (A11/Z4S)*A11 A120 = (A12/Z6S)*A12 AP110 = (AP11/Z5S)*AP11 AP120 = (AP12/Z7S)*AP12 G11 = GN*(AK11*FM1 - GAMMA*(ALFA*Z(4)/Z(1)+Z(5)/Z(2))) G12 = GN*((1.D0-FM1)/LAMN-GAMMA*(ALFA*Z(6)/Z(1)+Z(7)/Z(2))) EK11 = GAMMA*MASS*FAJ*(LAMP1*A11/ALP - MASS*AP11) EK12 = GAMMA*MASS*FAJ*(LAMP1*A12/ALP - MASS*AP12) DJ11 = EK11 + AK11*FJ2 -GM*(ALFA*A11/AN+AP11/ANP)*FJ1 DJ12 = EK12 + (FJ1-FJ2)/LAMN - GM*(ALFA*A12/AN+AP12/ANP)*FJ1 EK11 = EK11 - Z(11) EK12 = EK12 - Z(12) AZ8 = DJ11 - Z(8) AZ9 = DJ12 - Z(9) WRITE(6,43) A110,AP110,A120,AP120,AZ8,AZ9 ZM(1) = MASS ZA(1) = AN ZAP(1) = ANP ZG(1) = GN ZA11(1) = A11 ZA12(1) = A12 ZAP11(1) = AP11 ZAP12(1) = AP12 ZG11(1) = G11 ZG12(1) = G12 IF(KPASS.EQ.1) GO TO 210 206 WRITE(6,42) ANO,ANPO,AJO,GNO,AMASS A1 = A11 + LAM1*A12 AP1 = AP11 + LAM1*AP12 </pre> | <pre> BLST1590 BLST1600 BLST1610 BLST1620 BLST1630 BLST1640 BLST1650 BLST1660 BLST1670 BLST1680 BLST1690 BLST1700 BLST1710 BLST1720 BLST1730 BLST1740 BLST1750 BLST1760 BLST1770 BLST1780 BLST1790 BLST1800 BLST1810 BLST1820 BLST1830 BLST1840 BLST1850 BLST1860 BLST1870 BLST1880 BLST1890 BLST1900 BLST1910 BLST1920 BLST1930 BLST1940 BLST1950 BLST1960 BLST1970 BLST1980 BLST1990 BLST2000 BLST2010 BLST2020 BLST2030 BLST2040 BLST2050 BLST2060 BLST2070 BLST2080 BLST2090 BLST2100 BLST2110 BLST2120 </pre> |
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| A21 = Z(10) | BLST2130 |
| A22 = Z(12) | BLST2140 |
| AP21 = Z(11) | BLST2150 |
| AP22 = Z(13) | BLST2160 |
| A210 = (A21/Z10S)*A21 | BLST2170 |
| A220 = (A22/Z12S)*A22 | BLST2180 |
| AP210 = (AP21/Z11S)*AP21 | BLST2190 |
| AP220 = (AP22/Z13S)*AP22 | BLST2200 |
| G1 = G11 + LAM1*G12 | BLST2210 |
| EK21 = GAMMA*MASS*(FAJ*(A21*(LAMP1+LAMN) - ALP*MASS*AP21 + LAM1 | BLST2220 |
| 1*A1) + (LAMP1*A1 - ALP*MASS*AP1)**2/2.DO)/ALP | BLST2230 |
| DJ21 = EK21 + AK21*FJ3 - K1*RLAM*FJ2 + FJ1*((G1/GN)*(ALFA*A1/AN + | BLST2240 |
| 1 AP1/ANP) + ((G1/GN)**2 - RLAM**2 + GAMMA*(ALFA*(A1/AN)**2 + (AP1 | BLST2250 |
| 2/ANP)**2))/2.DO - GM*(ALFA*A21/AN+AP21/ANP) + ALFA*(A1/AN)*((ALFA | BLST2260 |
| 3 -1.DO)*A1/AN/2.DO+AP1/ANP)) | BLST2270 |
| EK22 = GAMMA*MASS*FAJ*(A22*(LAMP1+LAMN) - ALP*MASS*AP22)/ALP | BLST2280 |
| DJ22 = EK22 + (FJ1-FJ3)/LAMN/2.DO - GM*(ALFA*A22/AN+AP22/ANP)*FJ1 | BLST2290 |
| EK21 = EK21 - Z(8) | BLST2300 |
| EK22 = EK22 - Z(9) | BLST2310 |
| AZ14 = DJ21 - Z(14) | BLST2320 |
| AZ15 = DJ22 - Z(15) | BLST2330 |
| WRITE(6,44) A210,AP210,A220,AP220,AZ14,AZ15 | BLST2340 |
| ZA21(1) = A21 | BLST2350 |
| ZA22(1) = A22 | BLST2360 |
| ZAP21(1) = AP21 | BLST2370 |
| ZAP22(1) = AP22 | BLST2380 |
| ZG21(1) = (((G1/GN)**2 - RLAM**2)/2.DO - GAMMA*(ALFA*Z(10)/Z(1) | BLST2390 |
| 1 + Z(11)/Z(2) - (ALFA*(A1/Z(1))**2 + (AP1/Z(2))**2)/2.DO) | BLST2400 |
| 2 + AK21*FM1**2 - K1*RLAM*FM1)*GN | BLST2410 |
| ZG22(1) = ((1.DO-FM1**2)/2.DO/LAMN - GAMMA*(ALFA*Z(12)/Z(1) + | BLST2420 |
| 1 Z(13)/Z(2)))*GN | BLST2430 |
| IF(KPASS.EQ.2) GO TO 220 | BLST2440 |
| COMPUTE LAMEDAS AND ENERGY INTEGRAL CONTIBUTIONS | BLST2450 |
| 210 OMEGO = 2.DO/(LAMN+2.DO) | BLST2460 |
| OMEG1 = (2.DO*LAMN+2.DO)/(3.DO*LAMN+2.DO) | BLST2470 |
| LAM1 = (1.DO/GM/ALP-AZ8)/(AZ9+AJO*(BETA*OMEG1/OMEGO-ALP)/LAMN2) | BLST2480 |
| RLAM = LAM1/LAMN | BLST2490 |
| K1 = AK11 - RLAM | BLST2500 |
| A21 = AK11*RLAM - 2.DO/GMOG/GM - GMOG**2/8.DO - RLAM**2/2.DO | BLST2510 |
| AJ = AZ8 + LAM1*AZ9 | BLST2520 |
| EK1 = EK11 + LAM1*EK12 | BLST2530 |
| WRITE(6,47) DJ11,DJ12,LAM1,AJ1,EK1,K1 | BLST2540 |
| WRITE(6,46) RLAM,AK21 | BLST2550 |
| NV = 15 | BLST2560 |
| KPASS = 2 | BLST2570 |
| GO TO 20 | BLST2580 |
| 220 RZ1 = -LAM1/LAMN2 | BLST2590 |
| OMEG2 = (4.DO*LAMN+2.DO)/(5.DO*LAMN+2.DO) | BLST2600 |
| SA = (BETA*OMEG2/OMEGO - ALP)/2.DO | BLST2610 |
| SB = -AJO*SA/LAMN2 | BLST2620 |
| SA = AJO*BZ1**2*(SA*LAMP1 + BETA*(BETA-1.DO)*(OMEG1/OMEGO)**2 | BLST2630 |
| 1/2.DO + ALP*((ALFA+2.DO)/2.DO - BETA*OMEG1/OMEGO)) | BLST2640 |
| LAM2 = (SA - AZ14)/(AZ15 - SB) | BLST2650 |
| LAM4 = -(1.DO-BETA/ALP)*((ALFA+2.DO)/4.DO - LAM2 - 2.DO*LAM1 - | BLST2660 |

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|---|----------|
| 1 3.DO*LAMN) | BLST2670 |
| LAM3 = -LAMN - LAM1 - LAM2 - LAM4 | BLST2680 |
| AJ2 = AZ14 + LAM2*AZ15 | BLST2690 |
| EK2 = EK21 + LAM2*EK22 | BLST2700 |
| WRITE(6,48) DJ21,DJ22,LAM2,AJ2,EK2 | BLST2710 |
| COMPUTE SHCCK TRAJECTORY | BLST2720 |
| WRITE(6,40) | BLST2730 |
| BZ0 = ((2.DO+BETA)/(ALFA+3.DO))*BETA/AJ0**((1.DO/(ALP-BETA)) | BLST2740 |
| BZ(1) = BZ1 | BLST2750 |
| BZ(2) = -(LAM2 + LAMP1*LAM1*BZ1)/2.DO/LAMN2 | BLST2760 |
| BZ(3) = -(LAM3+(LAMP1+LAMN)*LAM1*BZ(2)+LAMP1*LAM2*BZ1)/3.DO/LAMN2 | BLST2770 |
| BZ(4) = -(LAM4+(1.DO+3.DO*LAMN)*LAM1*BZ(3)+(LAMP1+LAMN)*LAM2*BZ(2)+ | BLST2780 |
| LAMP1*LAM3*BZ1)/4.DO/LAMN2 | BLST2790 |
| DO 221 K=5,100 | BLST2800 |
| AK = K | BLST2810 |
| BZ(K) = -((1.DO+(AK-4.DO)*LAMN)*BZ(K-4)*LAM4 + (1.DO+(AK-3.DO) | BLST2820 |
| 1*LAMN)*BZ(K-3)*LAM3+(1.DO+(AK-2.DO)*LAMN)*BZ(K-2)*LAM2 | BLST2830 |
| 2 + (1.DO+(AK-1.DO)*LAMN)*BZ(K-1)*LAM1)/LAMN2/AK | BLST2840 |
| 221 OM(K) = 1.DO/(1.DO+.5DO/(AK+1.DO/LAMN)) | BLST2850 |
| OM(1) = OMEG1 | BLST2860 |
| OM(2) = OMEG2 | BLST2870 |
| OM(3) = 1.DO/(1.DO+.5DO/(3.DO+1.DO/LAMN)) | BLST2880 |
| OM(4) = 1.DO/(1.DO+.5DO/(4.DO+1.DO/LAMN)) | BLST2890 |
| Y = 0.DO | BLST2900 |
| DO 225 I=1,20 | BLST2910 |
| SUMZ = 0.DO | BLST2920 |
| SUM0 = 0.DO | BLST2930 |
| DO 222 K=1,100 | BLST2940 |
| TERMZ = BZ(K)*Y**K | BLST2950 |
| TERMO = OM(K)*TERMZ | BLST2960 |
| IF(TERMO.LT.1.D-5.AND.TERMZ.LT.1.D-5) GO TO 223 | BLST2970 |
| SUMZ = SUMZ + TERMZ | BLST2980 |
| 222 SUM0 = SUM0 + TERMO | BLST2990 |
| 223 FIRST = BZ0*Y**((1.DO/LAMN) | BLST3000 |
| BZS = (1.DO + SUMZ)*FIRST | BLST3010 |
| OMEGA = (OMEG0 + SUM0)*FIRST*DSORT(Y) | BLST3020 |
| LAMDA = LAMN + LAM1*Y + LAM2*Y**2 + LAM3*Y**3 + LAM4*Y**4 | BLST3030 |
| AJ = AJ0 + AJ1*Y + AJ2*Y**2 | BLST3040 |
| EK = EK0 + EK1*Y + EK2*Y**2 | BLST3050 |
| EFRAC = EK/AJ | BLST3060 |
| WRITE(6,440) Y,OMEGA,BZS,AJ,EFRAC,LAMDA | BLST3070 |
| 225 Y = Y + .05DO | BLST3080 |
| WRITE(6,442) ALFA,BETA,AJ0,AJ1,AJ2,LAMN,LAM1,LAM2,LAM3,LAM4, | BLST3090 |
| 1 AZ8,AZ9,AZ14,AZ15,GAMMA,SAVE | BLST3100 |
| COMPUTE FLOW FIELD | BLST3110 |
| IF(KFIELD.EQ.0) GO TO 10 | BLST3120 |
| DO 350 II=1,9 | BLST3130 |
| Y = 0.1DO*(II-1) | BLST3140 |
| WRITE(6,40) | BLST3150 |
| LAMDA = LAMN + LAM1*Y + LAM2*Y**2 + LAM3*Y**3 + LAM4*Y**4 | BLST3160 |
| WRITE(6,445) Y,LAMDA | BLST3170 |
| WRITE(6,446) | BLST3180 |
| DO 350 KK=1,110 | BLST3190 |
| IF(KK.EQ.1) GO TO 302 | BLST3200 |

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IF(MCD((KK-1),5).NE.0.AND.KK.LT.100) GO TO 350
IF (KK.LT.92) GO TO 302
IF(KK.GT.100) GO TO 301
JJ = 101 - (KK-91)*10
GO TO 303
301 JJ = 111 - KK
GO TO 303
302 JJ = 192 - KK
303 ZA1 = ZA11(JJ) + LAM1*ZA12(JJ)
   ZA2 = ZA21(JJ) + LAM2*ZA22(JJ)
   ZAP1 = ZAP11(JJ) + LAM1*ZAP12(JJ)
   ZAP2 = ZAP21(JJ) + LAM2*ZAP22(JJ)
   ZAT = ZA(JJ) + ZA1*Y + ZA2*Y**2
   ZAPT = ZAP(JJ) + ZAP1*Y + ZAP2*Y**2
   V = ALP*ZAT**L*ZAPT
   VEL = ZAT - ALP*ZM(JJ)*ZAPT + LAMDA*Y*(ZA1+2.DO*ZA2*Y)
   ZGT = ZG(JJ) + (ZG11(JJ) + LAM1*ZG12(JJ))*Y + (ZG21(JJ)+LAM2*
1 ZG22(JJ))*Y**2
   IF(KK.GT.1) GO TO 305
   ZGTF = ZGT
   VF = V
   VELF = VEL
305 RHO = VF/V
   ZGT = ZGT/ZGTF
   VELR = VEL/VELF
   WRITE(6,448) ZM(JJ),ZAT,ZGT,RHO,VELR
350 CONTINUE
   GO TO 10
440 FORMAT(' Y =',F6.3,' OMEGA =',E11.5,' Z =',E11.5,' J =',F10.5
1,' KE/J =',F10.5,' LAMDA =',F10.5)
442 FORMAT(/,' SUMMARY',/, ' ALFA=',F10.5,2X,' BETA=',F10.5,2X,' JO=',
1F10.5,2X,' J1=',F10.5,2X,' J2=',F10.5,/, ' LAMO=',F10.5,2X,' LAM1=',
2F10.5,2X,' LAM2=',F10.5,2X,' LAM3=',F10.5,2X,' LAM4=',F10.5,/, ' J11='
3,F10.5,2X,' J12=',F10.5,2X,' J21=',F10.5,2X,' J22=',F10.5,/, ' GAMMA='
4,F10.5,2X,' STEP=',F10.5)
445 FORMAT(/,' Y =',F5.2,2X,' LAMDA =',F10.5)
446 FORMAT(4X,15X,' MASS',13X,' RADIUS',11X,' PRESSURE',12X,' DENSITY'
1,11X,' VELOCITY',//)
448 FORMAT(4X,5F19.5)
999 STOP
END

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BLST3210
 BLST3220
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 BLST3250
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|--|-----|-----|
| SUBROUTINE FCT(Z,ZP,C,M) | FCT | 10 |
| IMPLICIT REAL*8 (A-H,O-Z) | FCT | 20 |
| REAL*8 LAMN,KN,M,LAM1,K1,LAMP1 | FCT | 30 |
| DIMENSION Z(15),ZP(15),C(16) | FCT | 40 |
| C AND Z TRANSFERS | FCT | 50 |
| IF(M.NE.1.DO) GO TO 10 | FCT | 60 |
| LAMN = C(1) | FCT | 70 |
| KN = C(2) | FCT | 80 |
| K1 = C(3) | FCT | 90 |
| ALP = C(4) | FCT | 100 |
| GAMM = C(5) | FCT | 110 |
| GMCG = C(6) | FCT | 120 |
| AK11 = C(7) | FCT | 130 |
| AK21 = C(8) | FCT | 140 |
| RLAM = C(10) | FCT | 150 |
| LAM1 = C(11) | FCT | 160 |
| ALF = C(12) | FCT | 170 |
| GAM = C(13) | FCT | 180 |
| LAMP1 = LAMN + 1.DO | FCT | 190 |
| L = ALF + .0100 | FCT | 200 |
| 10 AN = Z(1) | FCT | 210 |
| ANP = Z(2) | FCT | 220 |
| COMPUTE COMMON TERMS | FCT | 230 |
| ANL = AN**L | FCT | 240 |
| FM1 = M**((LAMN/ALP) | FCT | 250 |
| GN = KN/(ANL*ANP)**GAM/FM1 | FCT | 260 |
| FAJ = AN - ALP*M*ANP | FCT | 270 |
| AA = ALP*GAM*(ANL*GN/ANP - ALP*M**2) | FCT | 280 |
| COMPUTE ZERO ORDER DERIVATIVES | FCT | 290 |
| R = ALP*GAM*(2.DO*ALF + LAMN)*M/2.DO | FCT | 300 |
| CO = GAM*(LAMN/2.DO + GN*ALF*ALP*ANL*ANP/AN**2) | FCT | 310 |
| ZP(1) = ANP | FCT | 320 |
| ZP(2) = (8*ANP - CO*AN - GN*ANL*LAMN/M)/AA | FCT | 330 |
| EKO = GAM*FAJ**2/ALP/2.DO | FCT | 340 |
| ZP(3) = EKO + GN*ANL*ANP/GAMM | FCT | 350 |
| COMPUTE FIRST ORDER DERIVATIVES | FCT | 360 |
| A11 = Z(4) | FCT | 370 |
| AP11 = Z(5) | FCT | 380 |
| A12 = Z(6) | FCT | 390 |
| AP12 = Z(7) | FCT | 400 |
| ANPP = ZP(2) | FCT | 410 |
| GNP = -GN*(LAMN/ALP/M + GAM*(ALF*ANP/AN+ANPP/ANP)) | FCT | 420 |
| BB = ALP*GAM*(ANL*(GN*ANPP/ANP**2 - GNP/ANP - ALF*GN/AN) | FCT | 430 |
| 1 -2.DO*LAMN*M) + R | FCT | 440 |
| CC = CO + GAM*LAMN**2/2.DO - GAMM*ALF*ALP*ANL*GNP/AN | FCT | 450 |
| DD = AK11*FM1*ANL*(GNP*ALP + GN*LAMN/M) | FCT | 460 |
| EE = (ANL*ALF*GNP - DD/AK11)/LAMN - FAJ*GAM/2.DO | FCT | 470 |
| ZP(4) = AP11 | FCT | 480 |
| ZP(5) = (8R*AP11+CC*A11+DD)/AA | FCT | 490 |
| ZP(6) = AP12 | FCT | 500 |
| ZP(7) = (8R*AP12+CC*A12+EE)/AA | FCT | 510 |
| IF(K1.NE.0.DO) GO TO 100 | FCT | 520 |
| EK11 = GAM*FAJ*(A11*LAMP1-ALP*M*AP11)/ALP | FCT | 530 |
| EK12 = GAM*FAJ*(A12*LAMP1-ALP*M*AP12)/ALP | FCT | 540 |

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| ZP(8) = EK11 + GN*ANL*(AK11*ANP*FM1/GAMM-AP11-ALF*ANP*A11/AN) | FCT 550 |
| ZP(9) = EK12 + GN*ANL*(ANP*(1.00-FM1)/GAMM/LAMN - AP12 | FCT 560 |
| 1 - ALF*ANP*A12/AN) | FCT 570 |
| ZP(10) = EKO | FCT 580 |
| ZP(11) = EK11 | FCT 590 |
| ZP(12) = EK12 | FCT 600 |
| RETURN | FCT 610 |
| COMPUTE SECOND ORDER DERIVATIVES | FCT 620 |
| 100 A1 = A11 + LAM1*A12 | FCT 630 |
| AP1 = AP11 + LAM1*AP12 | FCT 640 |
| APP1 = ZP(5) + LAM1*ZP(7) | FCT 650 |
| G1 = GN*(RLAM - GAM*(ALF*A1/AN + AP1/ANP) + K1*FM1) | FCT 660 |
| GP1 = G1*GNP/GN + GN*(K1*LAMN*FM1/M/ALP - GAM*(ALF*(AP1/AN-A1 | FCT 670 |
| 1*ANP/AN**2) + (APP1/ANP-AP1*ANPP/ANP**2))) | FCT 680 |
| A21 = Z(10) | FCT 690 |
| AP21 = Z(11) | FCT 700 |
| A22 = Z(12) | FCT 710 |
| AP22 = Z(13) | FCT 720 |
| BBB = BB - 2.00*ALP*GAM*M*LAMN | FCT 730 |
| CCC = CC + GAM*LAMN*(1.00+2.500*LAMN) | FCT 740 |
| DDD = AK21*LAMN*FM1**2*(GNP/LAMN+2.00*GN/ALP/M) - K1*LAM1*FM1* | FCT 750 |
| 1(GNP/LAMN+GN/ALP/M) + GNP*(GAM*(ALF*(A1/AN)**2+(AP1/ANP)**2) - | FCT 760 |
| 2(G1/GN)**2-RLAM**2)/2.00+GP1*(G1/GN+ALF*A1/AN)+GAM*GN*(ALF*A1* | FCT 770 |
| 3(AN*AP1-A1*ANP)/AN**3 + (ANP*AP1*APP1-AP1**2*ANPP)/ANP**3) | FCT 780 |
| DDD = DDD*ALP*ANL + GAM*LAM1*(A1*(1.00+4.00*LAMN) - 3.00*ALP*M* | FCT 790 |
| 1 AP1)/2.00 + ALP*ALF*(ALF-1.00)*A1**2*GNP/2.00 | FCT 800 |
| EFE = .500*ALP*(ANL*GNP*(1.00-FM1**2)/LAMN + GAM*M*ANP) - | FCT 810 |
| 1 ANL*GN*FM1**2/M - GAM*AN/2.00 | FCT 820 |
| ZP(10) = AP21 | FCT 830 |
| ZP(11) = (BBB*AP21+CCC*A21+DDD)/AA | FCT 840 |
| ZP(12) = AP22 | FCT 850 |
| ZP(13) = (BBB*AP22+CCC*A22+EFE)/AA | FCT 860 |
| G21 = (((G1/GN)**2-RLAM**2)/2.00 + AK21*FM1**2 - RLAM*K1*FM1 | FCT 870 |
| 1 - GAM*(ALF*(A21/AN-(A1/AN)**2/2.00) + AP21/ANP - (AP1/ANP)**2 | FCT 880 |
| 2/2.00))*GN | FCT 890 |
| G22 = GN*((1.00-FM1**2)/LAMN/2.00 - GAM*(ALF*A22/AN+AP22/ANP)) | FCT 900 |
| EK21 = GAM*(FAJ*(LAM1*A1 + (LAMP1+LAMN)*A21 - ALP*M*AP21) | FCT 910 |
| 1 + (LAMP1*A1 - ALP*M*AP1)**2/2.00)/ALP | FCT 920 |
| ZP(14) = EK21 + ANL*ANP*((ALF*A21/AN + AP21/ANP + (AP1/ANP+(ALF | FCT 930 |
| 1-1.00)*A1/AN/2.00)*ALF*A1/AN)*GN+G1*(AP1/ANP+ALF*A1/AN)+G21)/GAMM | FCT 940 |
| EK22 = GAM*(FAJ*((LAMP1+LAMN)*A22 - ALP*M*AP22))/ALP | FCT 950 |
| ZP(15) = EK22 + ANL*ANP*(GN*(ALF*A22/AN+AP22/ANP) + G22)/GAMM | FCT 960 |
| ZP(8) = EK21 | FCT 970 |
| ZP(9) = EK22 | FCT 980 |
| RETURN | FCT 990 |
| END | FCT 1000 |

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SUBROUTINE RUNKUT(Z,X,C,H,N)
CONSTANT STEP, FOURTH ORDER RUNGE-KUTTA INTEGRATION ALGORITHM
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION Z(15),C(16),Y(15),RK1(15),RK2(15),RK3(15),RK4(15)
H2 = H/2.00
CALL FCT(Z,RK1,C,X)
DO 10 J=1,N
10 Y(J) = Z(J) + H2*RK1(J)
X = X+ H2
CALL FCT(Y,RK2,C,X)
DO 11 J=1,N
11 Y(J) = Z(J) + H2*RK2(J)
CALL FCT(Y,RK3,C,X)
DO 12 J=1,N
12 Y(J) = Z(J) + H *RK3(J)
X = X+ H2
CALL FCT(Y,RK4,C,X)
DO 13 J=1,N
13 Z(J) = Z(J) + H*(RK1(J)+2.00*(RK2(J)+RK3(J))+RK4(J))/6.00
RETURN
END

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RNKT 10
RNKT 20
RNKT 30
RNKT 40
RNKT 50
RNKT 60
RNKT 70
RNKT 80
RNKT 90
RNKT 100
RNKT 110
RNKT 120
RNKT 130
RNKT 140
RNKT 150
RNKT 160
RNKT 170
RNKT 180
RNKT 190
RNKT 200
RNKT 210

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